Photons' in Relativistic Plasma with Velocity Shear: A novel mechanism for power law spectra at high energies

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HEPRO - VIII, October, 23rd, 2023
Overview

- **Introduction**: Fermi acceleration and photon energy gain in shearing flows
- **Physical mechanism** of photon energy gain
- Generation of power law spectrum at high energies
- **Results**: Comparison of estimated spectral slopes with Monte Carlo Simulations
- **Conclusions**: Significance of the work
First order Fermi acceleration:
Shock acceleration of charged particles

Electrons gain energy and produce a power law spectrum when they interact with regions having different speeds in the plasma.
Particle acceleration in shearing flows

But what about a photon, trapped and escapes from shearing scatters??
We need investigation and a clear picture of this process.
Any relativistic flow with a presence of
(i) Shear,
(ii) High optical depth
would lead to net photon energy gain producing a power law emission of spectra
Generation of power law due to *Velocity shear* in the plasma

Physical picture

Simulations of radiation-driven winds from Keplerian discs (Raychaudhuri, Vyas & Chattopadhyay, 2021)
Attempts to explain high energy power law

- **Synchrotron shock model**
  - Tavani 1996; Cohen et al 1997; Panaitescu et al. 1999; Frontera et al. 2000

- **Backscattering dominated emission**

- **Bulk Comptonization from structured jets with photospheric emission**
  - Lundman et al. 2013, Vyas & Pe’er 2022 (current work)

*Piron, Comptes Rendus Physique 17, no. 6 (2016): 617-631*
Photon energy gain from relativistic jets with structured Lorentz factor profile

\[ \Gamma(\theta) = \Gamma_{\text{min}} + \frac{\Gamma_0}{\sqrt{\left(\frac{\theta}{\theta_j}\right)^{2p} + 1}} \]

Lorentz factor (\(\Gamma\)) as a function of \(\theta\)

Lundman et al. 2013, 428-3, 2430
Lundman et. al. 2014, MNRAS 440 3292

Case of GRB jets: Energy gain in scattering process

1. Analytic estimates of spectral slopes

\[
g = \frac{\varepsilon_2}{\varepsilon} = \frac{1 - \beta_2 \cos(\theta_s - \theta_{el})}{1 - \beta_2 \cos(\theta_2)}
\]

\[
\frac{\Gamma_2}{\Gamma} = 1 + \frac{\delta \Gamma}{\Gamma} = 1 + \sum \frac{\partial \log \Gamma}{\partial x^i} \delta x^i
\]
Energy gain in scattering process

If the local mean free path measured in the lab frame is \( \lambda(r, \theta) \), then

\[
\tan \theta_{el} = \frac{\lambda \sin \theta_s}{r + \lambda \cos \theta_s} = \frac{a \theta_s}{1 + a \cos \theta_s}
\]

Taylor expansion allows us to estimate the gain as:

\[
g(r, \theta) \approx \frac{1}{2} \left[ 1 + \left[ 1 + \sum \frac{\partial \log \Gamma}{\partial x^i} \delta x^i \right]^2 \frac{1}{(1 + a)^2} \right]
\]

\[ a \equiv \lambda/r \]
Averaged gain over all scatterings

The expectation value of the photon energy gain in the plasma is evaluated by integrating the average gain over the entire region of scattering

\[ \bar{g} = \frac{1}{V} \int_V g_a(\theta, r) dV. \]
Escape probabilities for the photon from accelerating region

\[ P_e(r, \theta) = \exp[-\tau(r, \theta)] \]

Escape probability at location \( r, \theta \)

\[ \bar{\tau} = \frac{r_{ph}}{r} \]

Photospheric optical depth

\[ \tau_2 = \int_0^{s_0} \Gamma(1 - \nu \cos \theta_s)n'_e \sigma ds \]

Angular optical depth

\[ \tau = \min(\tau_1, \tau_2) \]

\[ P(r, \theta) = 1 - \exp[-\tau(r, \theta)] \]

Probability for next scattering at location \( r, \theta \) without escape

Figure 2. Lorentz factor (\( \Gamma \)) profile of the jet characterized by equation 13 with parameters \( p = 2.0, \theta_i = 0.01 \) rad, \( \Gamma_0 = 100 \) and \( \Gamma_{\min} = 1.2, \theta_\ast = \theta_\ast \Gamma_0^{1/p} \). The inner jet region is for \( \theta < \theta_\ast \) while outer region extends beyond \( \theta_\ast \). The region bounded within \( \theta_i - \theta_\ast \) harbours an effective velocity shear leading to photon energy gain.
Averaged probabilities from the accelerating region

The average probability of the photon to have a scattering without escape within the jet is the probability $P(r, \theta)$ averaged over the available volume $V$ of scattering where velocity shear is present, i.e.,

$$\bar{P} = \frac{1}{V} \int_V P(r, \theta) dV.$$
Generation of power law spectrum at high energies

After scattering \( k \) times, there are \( N \) photons left out of \( N_0 \) photons in the accelerating region while \( N_0 - N \) have escaped,

\[
\frac{N}{N_0} = \tilde{P}^k \quad \text{or} \quad \ln \frac{N}{N_0} = k \ln \tilde{P}
\]

After \( k \) cycles, the photon’s energy is

\[
\varepsilon_k = \varepsilon_i \tilde{g}^k \quad \text{or} \quad \ln \frac{\varepsilon_k}{\varepsilon_i} = k \ln \tilde{g}
\]
Generation of power-law shaped spectra at high energies

\[
\ln \frac{N}{N_0} = \ln \frac{\varepsilon_k}{\varepsilon_i} \cdot \ln \bar{P} \cdot \ln \bar{g}
\]

\[
\frac{N}{N_0} = \left( \frac{\varepsilon_k}{\varepsilon_i} \right)^{\beta'}
\]

\[
\beta' = \frac{\ln \bar{P}}{\ln \bar{g}}
\]

\[
\frac{dN}{d\varepsilon_1} = N_0 \beta' \varepsilon_1^{\beta' - 1}
\]

\[
\beta = \beta' - 1 = \frac{\ln \bar{P}}{\ln \bar{g}} - 1
\]

We supply \( P \) and \( g \) for calculating the spectral slopes at high energies.
2. Numerical simulations

Numerical Code:

- The numerical simulations are based upon Monte Carlo method.
- We inject around 6 million photons deep inside the jet. Each of the photon goes through multiple scattering within the shear layers before it escapes.
- The escaped photons’ population is distributed then into bins of observing angles as well as energies.
- These binned photons produce the observed light curves as well as the spectrum for the given parameters.
Results: 
Simulated spectrum 
(On axis observer)

\[ \Gamma(\theta) = \Gamma_{\text{min}} + \frac{\Gamma_0}{\sqrt{\left(\frac{\theta}{\theta_\text{i}}\right)^{2p} + 1}} \]

Figure 2. Lorentz factor (Γ) profile of the jet characterized by equation 13 with parameters \( p = 2.0, \theta_\text{i} = 0.01 \text{ rad}, \Gamma_0 = 100 \) and \( \Gamma_{\text{min}} = 1.2, \theta_\text{e} = \theta_\text{i}\Gamma_0^{1/p} \). The inner jet region is for \( \theta < \theta_\text{i} \) while outer region extends beyond \( \theta_\text{e} \). The region bounded within \( \theta_\text{i} - \theta_\text{e} \) harbours an effective velocity shear leading to photon energy gain.

High energy tail produced by scattering within the shear layers
Simulated Spectra: Continued
Results: Analytic slopes and comparison with Monte Carlo Simulations

\[ \beta = \beta' - 1 = \frac{\ln \bar{P}}{\ln \bar{g}} - 1 \]

- Asymptotic slopes reach at -1.5 as \( p \to \infty \)
- For small and finite values of \( p \ (<2) \), the spectra vanishes due to geometric expansion of the jet
- The obtained range of \( \beta \) is compatible with the observed values
There are several other possible cases where such mechanism takes place

- A fast rotating star and its layers at the outskirts
- Relativistic turbulent plasma where there is a sharp velocity gradient in the turbulent eddies
- Sharply accelerating or decelerating relativistic jets/winds
Conclusions and Significance

Such power laws are capable of explaining high energy tails observed in GRB prompt phase spectra where $\beta$ ranges between $-3$ and $-1.5$. We found the theoretical range $-\infty$ to $-1.5$.

Inversely, we show that using the observed values of $\beta$, we can directly constraint the jet structure of these bursts.

The mechanism is important for emission from other such objects like AGNs or X-ray binary jets, fast spinning compact stars, accretion discs with velocity gradient etc. We will explore these possibilities in future.

Vyas and Pe’er, ApJL 943-1, L3, 7