Supercritical accretion disks with winds in General Relativity

Daniela Pérez[1], Gustavo E. Romero[1,2], Hirí Horák[3]

[1]Instituto Argentino de Radioastronomía (CONICET-CIC-UNLP)

[2]Facultad de Ciencias Astronómicas y Geofísicas de La Plata (UNLP)

[3]Astronomický ústav AV ČR

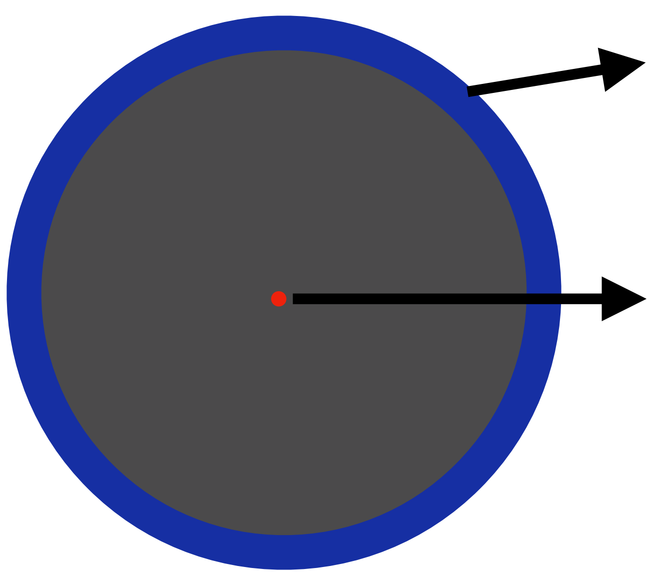
GARRA
Grupo de Astrofísica Relativista y Radioastronomía

Instituto Argentino de Radioastronomía

HEPRO VIII, October 26th 2023, Paris, France

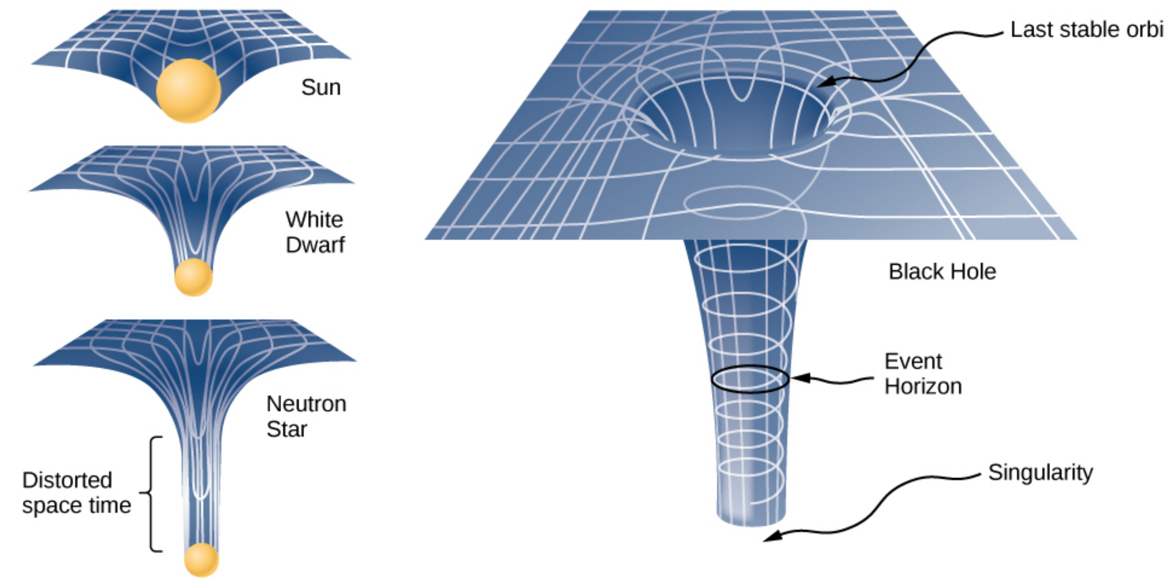
A **black hole** is essentially a spacetime region with a particular curvature.

Schwarzschild black hole: M
Kerr black hole: M, a



Event horizon $r_s = \dfrac{2GM}{c^2}$

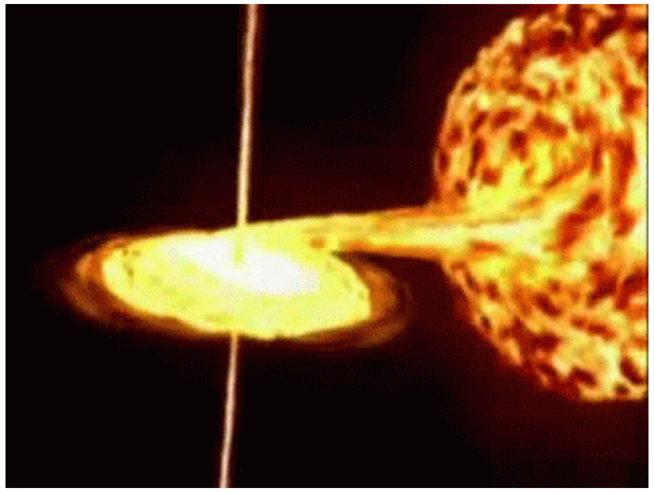
Singularity $r = 0$



Gravitational energy is efficiently transformed in other forms of energy

Zel’dovich and Novikov (1964)
Salpeter (1964)

Accretion is the process of matter falling into the potential well of a gravitating object. The accretion of matter with angular momentum can lead to the formation of an accretion disk around a compact object, such as a **black hole**.



- Key parameter in accretion disk theory

↓

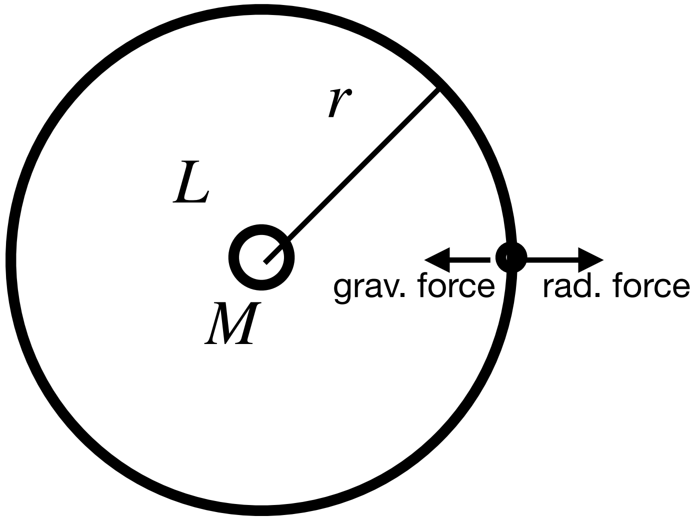
Critical mass accretion rate

$$\dot{M}_{\rm crit} \equiv \eta \dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{c^2} \approx 1.4 \times 10^{17} \frac{M}{M_\odot} {\rm g\ s^{-1}}$$

$L_{\rm Edd}$ **: Eddington luminosity**

M**: black hole mass**

The **Eddington luminosity** is the maximum luminosity that can be achieved when there is a balance between the force of radiation acting outward and the gravitational force acting inward.

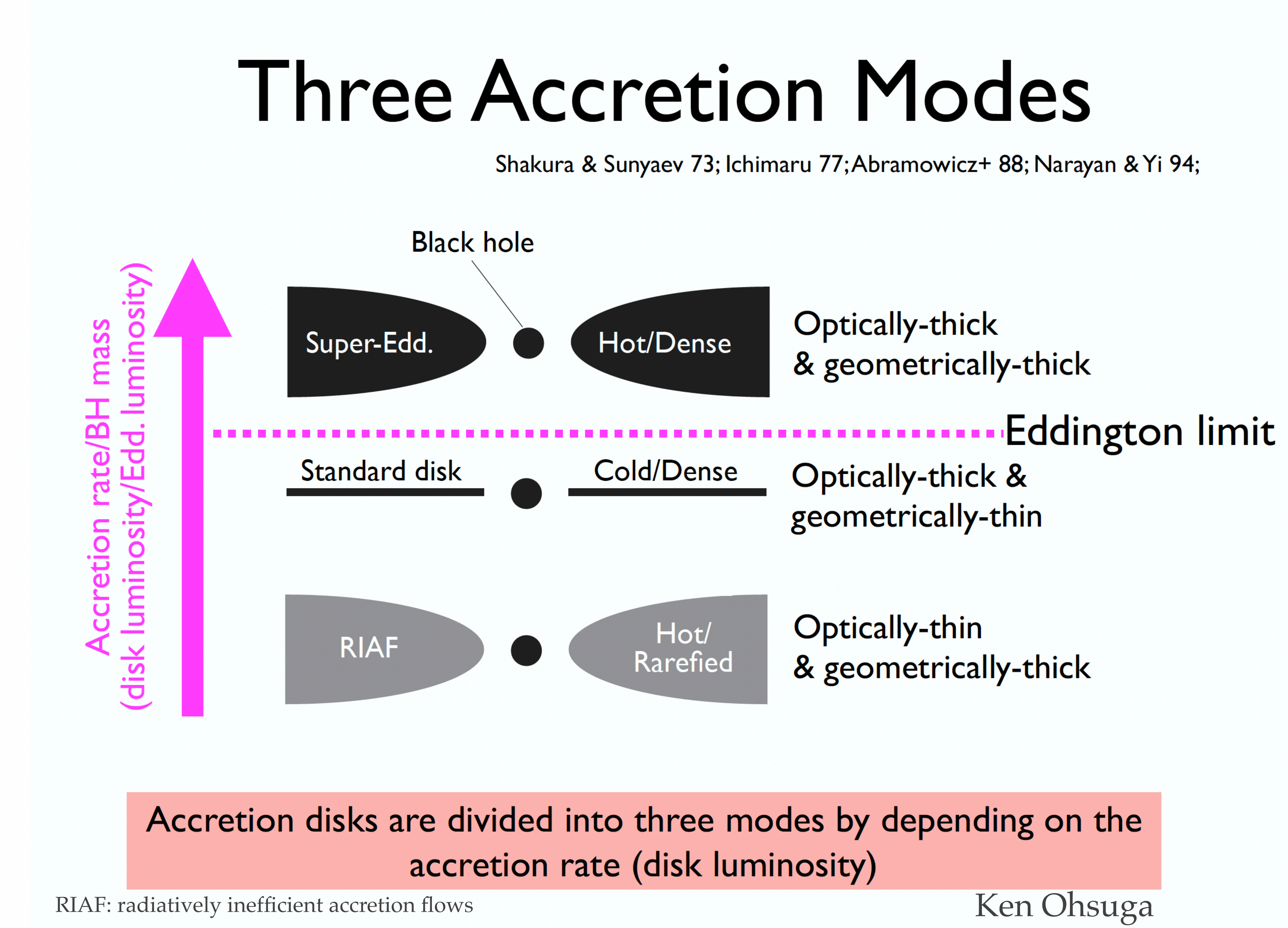


Depending on how the mass and critical accretion rate are related, we can define 3 different accretion regimes:

- $\dot{M} \ll \dot{M}_{\rm crit}$: geometrically thick and optically thin (Narayan and Yi, 1994)

- $\dot{M} \lesssim \dot{M}_{\rm crit}$: geometrically thin and optically thick (Shakura and Sunyaev, 1973; Ichimaru 77)

- $\dot{M} \gg \dot{M}_{\rm crit}$: geometrically and optically thick (Abramowicz et al. 1988, Fukue 2004)



Supercritical accretion disks with outflows

An important feature of supercritical accretion disks is the presence of winds.

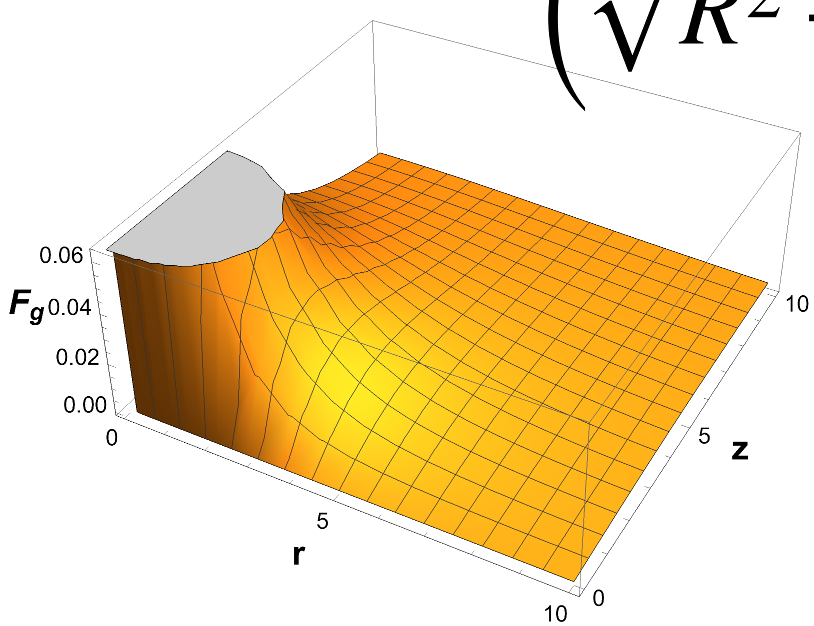
Assumptions:

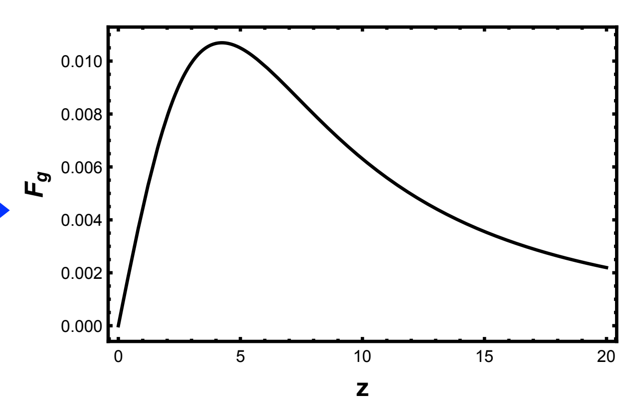
Fukue (2004)

- Radiation-pressure dominated standard accretion disk.
- Newtonian gravity. We employ cylindrical coordinates (R, ϕ, z), where $r = \sqrt{R^2 + z^2}$.
- Local force balance in the vertical direction:

$$\frac{GMz}{\left(\sqrt{R^2+z^2}\right)^3} = \frac{\sigma_T F}{m_p c} = \frac{3\sigma_T}{8\pi m_p c}\frac{GM\dot{M}}{R^3}$$

Radiative flux in the vertical direction





The vertical force in the vertical direction becomes a maximum at $z_{\max} = \frac{\sqrt{2}}{2}R$.

$$\underbrace{\frac{GMz_{\max}}{\left(\sqrt{R^2+z_{\max}^2}\right)^3}} = \underbrace{\frac{3\sigma_T}{8\pi m_p c}\frac{GM\dot{M}}{R^3}}$$

$$F_g|_{\max} = F_{\mathrm{rad}}$$

$$\downarrow$$

$$R_{\mathrm{crit}} = \frac{9\sqrt{3}}{16\pi}\frac{\sigma_T}{m_p c}\dot{M}_{\mathrm{input}}$$

For $R < R_{\mathrm{crit}}$, then $F_{\mathrm{g}}|_{\max} < F_{\max}$: the radiation pressure overcomes the gravitational force, and there will be mass loss in the form of outflows.

A model of supercritical accretion disks with winds in General Relativity (GR)

Assumptions:

- Kerr metric in cylindrical coordinates (t, r, z, ϕ) expanded up to second order in powers of z (Riffert and Herold, 1995).
- The energy-momentum tensor of the matter of the disk corresponds to a single-component fluid and also includes viscosity:

$$T^{\mu\nu} = \left(\rho + p\right) u^{\mu} u^{\nu} + p g^{\mu\nu} - t^{\mu\nu} \,.$$

ρ: energy density of the fluid
p: isotropic pressure
$t^{\mu\nu}$: viscous stress tensor
$u^{\mu} = (u^t, u^r, u^z, u^{\phi})$: four-velocity of the fluid

Conservation laws for matter

✓ Mass conservation or continuity equation → $\boxed{\left(\rho u^{\mu}\right)_{;\mu} = 0}$

✓ Conservation of the energy-momentum tensor

$$\boxed{\nabla_{\mu} T^{\mu}{}_{\nu} = \phi_{\nu}}$$

ϕ_{ν} Radiation four-force density

→ Radial momentum equation $\left(\delta^{\nu}{}_{r} + u^{\nu} u_{r}\right) \nabla_{\mu} T^{\mu}{}_{\nu} = h^{\nu}{}_{r} \phi_{\nu}$

→ Conservation of the angular momentum $\left(T^{\nu}{}_{\mu} \xi^{\mu}{}_{\phi}\right)_{;} = 0, \quad \Rightarrow \quad \left(T^{\nu}{}_{\phi}\right)_{;} = \phi_{\phi}$

→ Energy conservation $u^{\nu} \nabla_{\mu} T^{\mu}{}_{\nu} = u_{\mu} \phi^{\mu}$

→ Equation of motion in the vertical direction $\left(\delta^{\nu}{}_{z} + u^{\nu} u_{z}\right) \nabla_{\mu} T^{\mu}{}_{\nu} = \phi_{z}$

✓ Mass conservation or continuity equation

$$\boxed{\dot{M}(r) = \dot{M}_{input} - \dot{M}_w}$$

$$\dot{M}_w = \int_r^{r_{out}} 2\pi r\ \dot{m}_w dr$$

$$\dot{m}_w = \rho u^z \longrightarrow$$

Mass loss rate in outflows from the unit area of the disk surface.

✓ Radial momentum equation

$$\boxed{h^\nu{}_r \nabla_\mu T^\mu{}_\nu = a_r \left(p + \epsilon\right) + p_{,r} + u^r u_r p_{,r} + u^z u_r p_{,z} = h^\nu{}_r \phi_\nu}$$

✓ Conservation of the angular momentum

$$\boxed{\frac{\partial}{\partial r}\left[-\frac{\dot{M}}{2\pi}\ell - \sqrt{-g}\,W^r{}_\phi\right] + 2\sqrt{-g}\,\ell\ \dot{m}_w = \phi_\phi}.$$

$$\ell \equiv u_\phi$$

$$\int_{-h}^{h} t^r{}_\phi dz = W^r{}_\phi$$

✓ **Energy conservation**

$$\boxed{-\frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}\left[\left(p+\epsilon\right)\sqrt{-g}u^r\right]\underbrace{-\frac{1}{\sqrt{-g}}\frac{\partial}{\partial z}\left[\left(p+\epsilon\right)\sqrt{-g}u^z\right]}_{Q_w}+u^rp_{,r}+u^zp_{,z}\underbrace{-t^r{}_\phi\frac{d\Omega}{dr}u^t}_{Q^+}=u_\mu\phi^\mu}$$

✓ **Equation of motion in the vertical direction**

$$\boxed{h^\nu{}_z\nabla_\mu T^\mu{}_\nu = a_z\left(p+\epsilon\right)+p_{,z}+u^ru_z\,p_{,r}+u^zu_z\,p_{,z}-u^\phi u_z\nabla_r t^r_\phi=\phi_z}$$

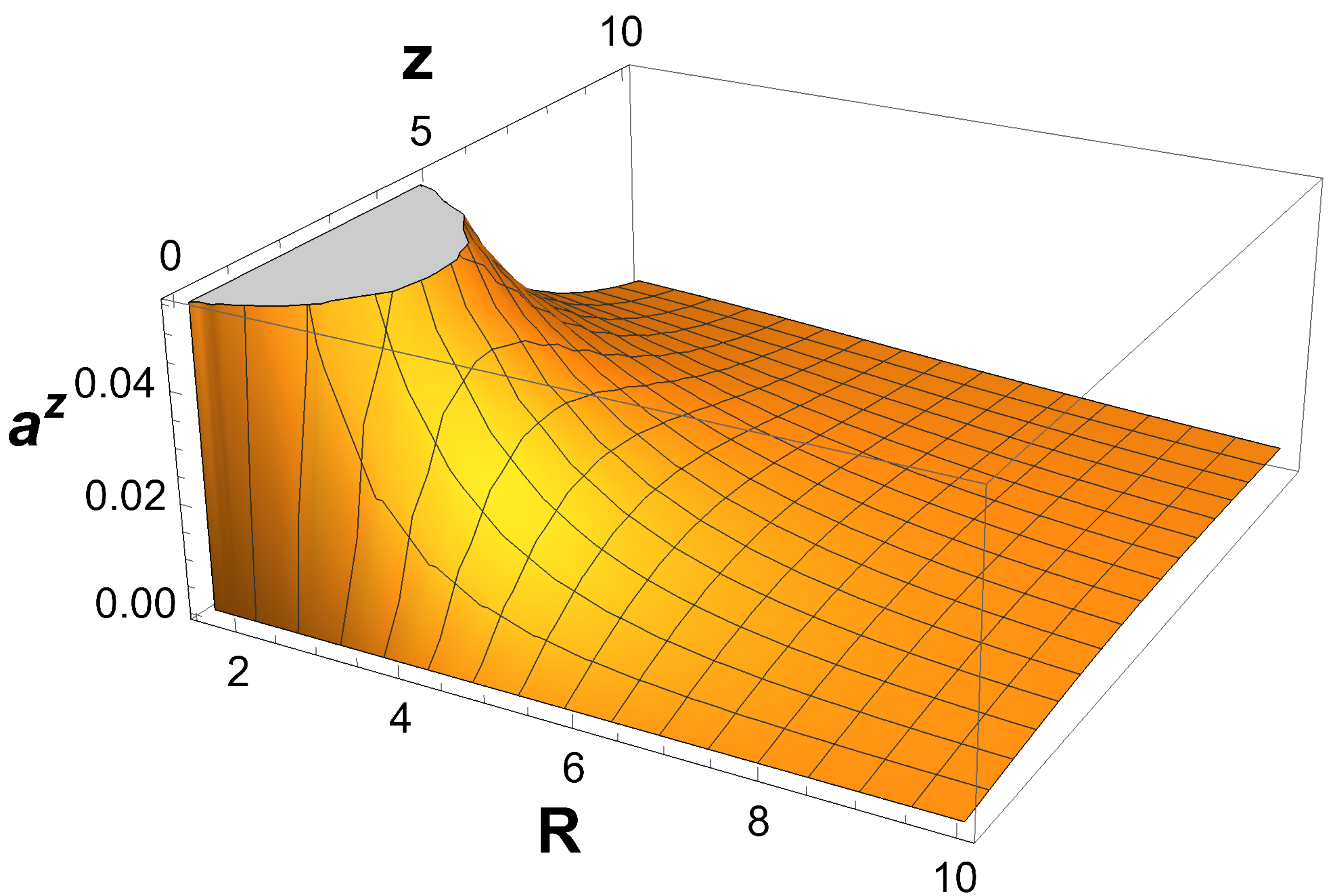
$$a_z = u^r\,u_{z,r}+\frac{1}{2}\frac{\partial}{\partial z}\left(u^zu_z\right)-\frac{1}{2}\left[g_{tt,z}\left(u^t\right)^2+2g_{t\phi,z}u^tu^\phi+g_{rr,z}\left(u^r\right)^2+g_{\phi\phi,z}\left(u^\phi\right)^2\right]$$

Treatment of the vertical motion of the fluid in GR

Vertical acceleration of gravity in Kerr spacetime

$$a^z = \frac{Mz}{r^3}\frac{C}{B}$$

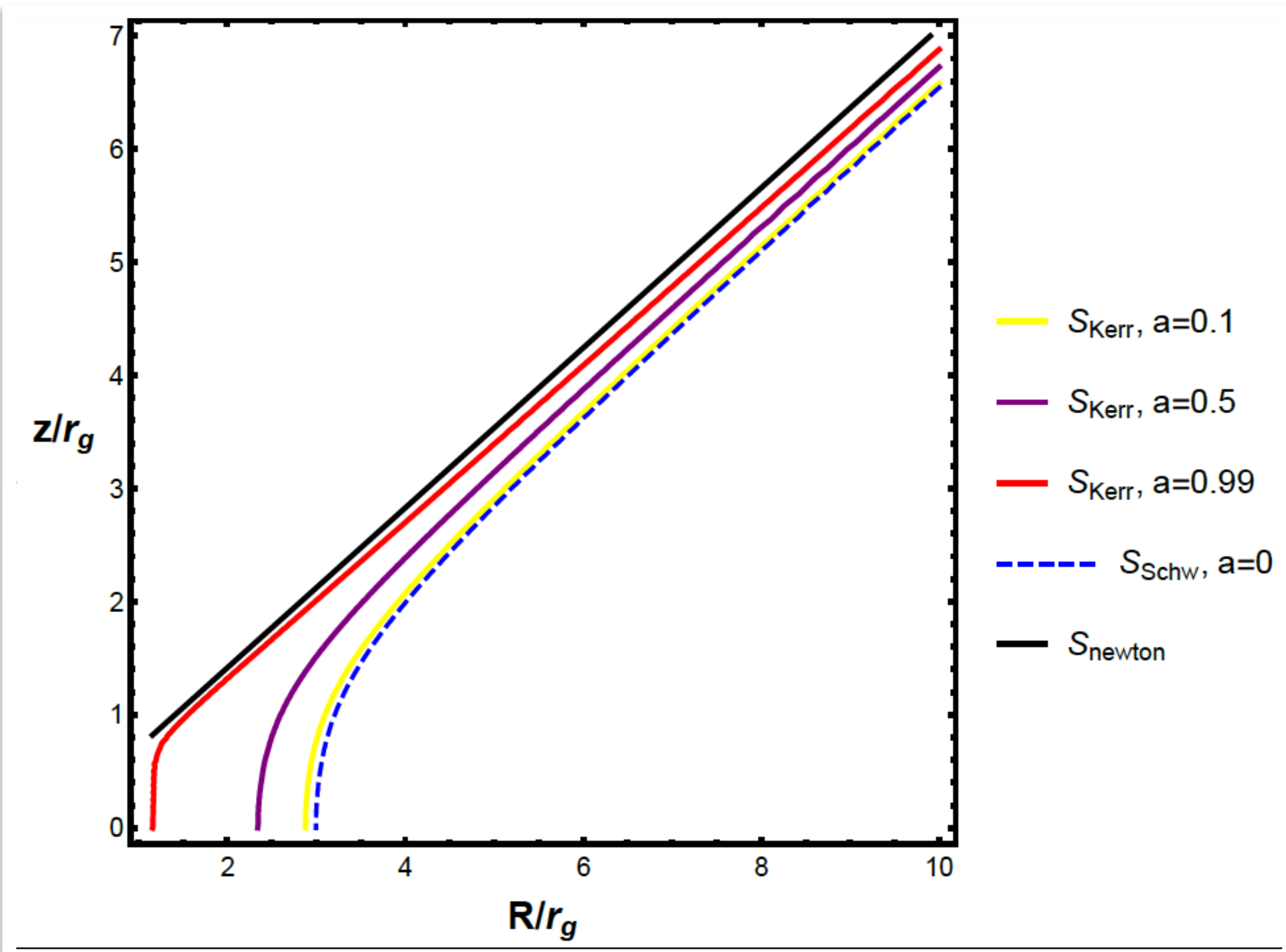
$$\begin{cases} C = 1 - 4\dfrac{a\sqrt{M}}{r^{3/2}} + 3\dfrac{a^2}{r^2} \\ B = 1 - \dfrac{3M}{r} + \dfrac{2a\sqrt{M}}{r^{3/2}} \end{cases}$$



If the value of the radiation flux is larger than the gravitational acceleration locally, the disk is not in hydrostatical balance.

The half thickness of the disk is defined as the value of the z coordinate where the acceleration of gravity is maximum.

Cao and Gu (2022)



✓ Equation of motion in the vertical direction

<u>Assumptions</u>

- $u^r \approx 0.$
- Radiation-pressure dominated accretion disk.

$$\frac{du^z}{dz} = \left[-\frac{p_{,z}}{\epsilon} \left(1 + g_{zz}(u^z)^2 \right) - \frac{1}{2} g_{zz,z}(u^z)^2 - a_{\mathrm{grav}} \right] \frac{1}{g_{zz}u^z}$$

$\propto$

$f_{\mathrm{rad}}(z)$: Radiation flux in the vertical direction

$$\frac{1}{\epsilon} p_{,z} = -\frac{\kappa_{\mathrm{T}}}{c} f_{\mathrm{rad}}(z)$$

κ_{T} Electron scattering

$$f_{\mathrm{rad}}[\tau(z)] = \frac{\tau(z)}{\tau(H)} \xi(r) f_{\mathrm{rad}}^{\mathrm{crit}}$$

Optical depth

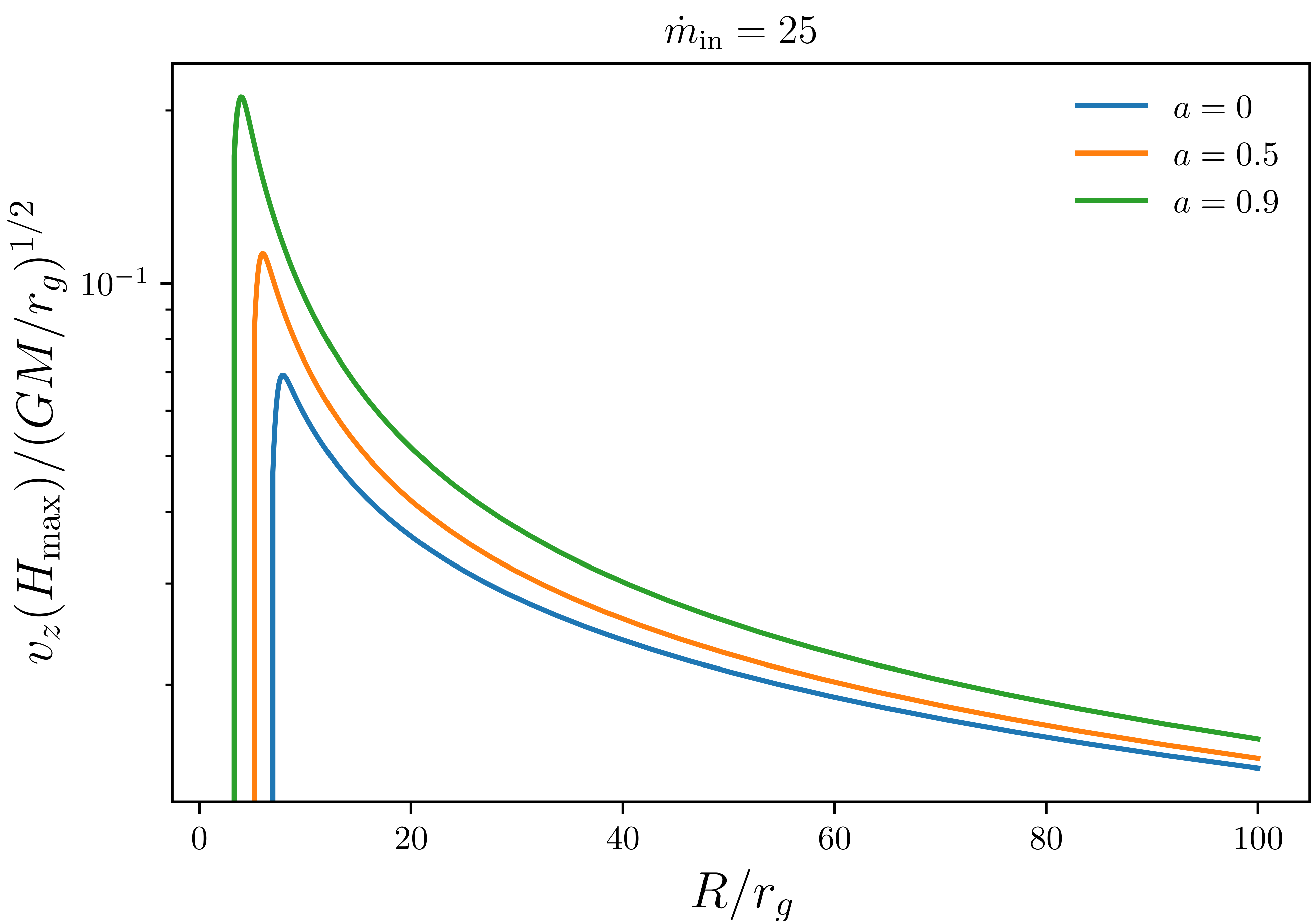
$$\tau(z) = \int_0^z \rho(z')\kappa_{\mathrm{T}} \sqrt{g_{zz}} dz'$$

Larger than the maximum vertical gravitational acceleration at that point.

Assumptions of the model

$$\dot{M}_{\rm in} = \frac{16\ \pi\ m_p\ c}{9\sqrt{3}\ \sigma_T} R_{\rm in} \longrightarrow \dot{m}_{\rm in} \equiv \frac{\dot{M}_{\rm input}}{\dot{M}_{\rm crit}} \simeq 0.25 \left(\frac{R_{\rm in}}{R_{\rm g}}\right)$$

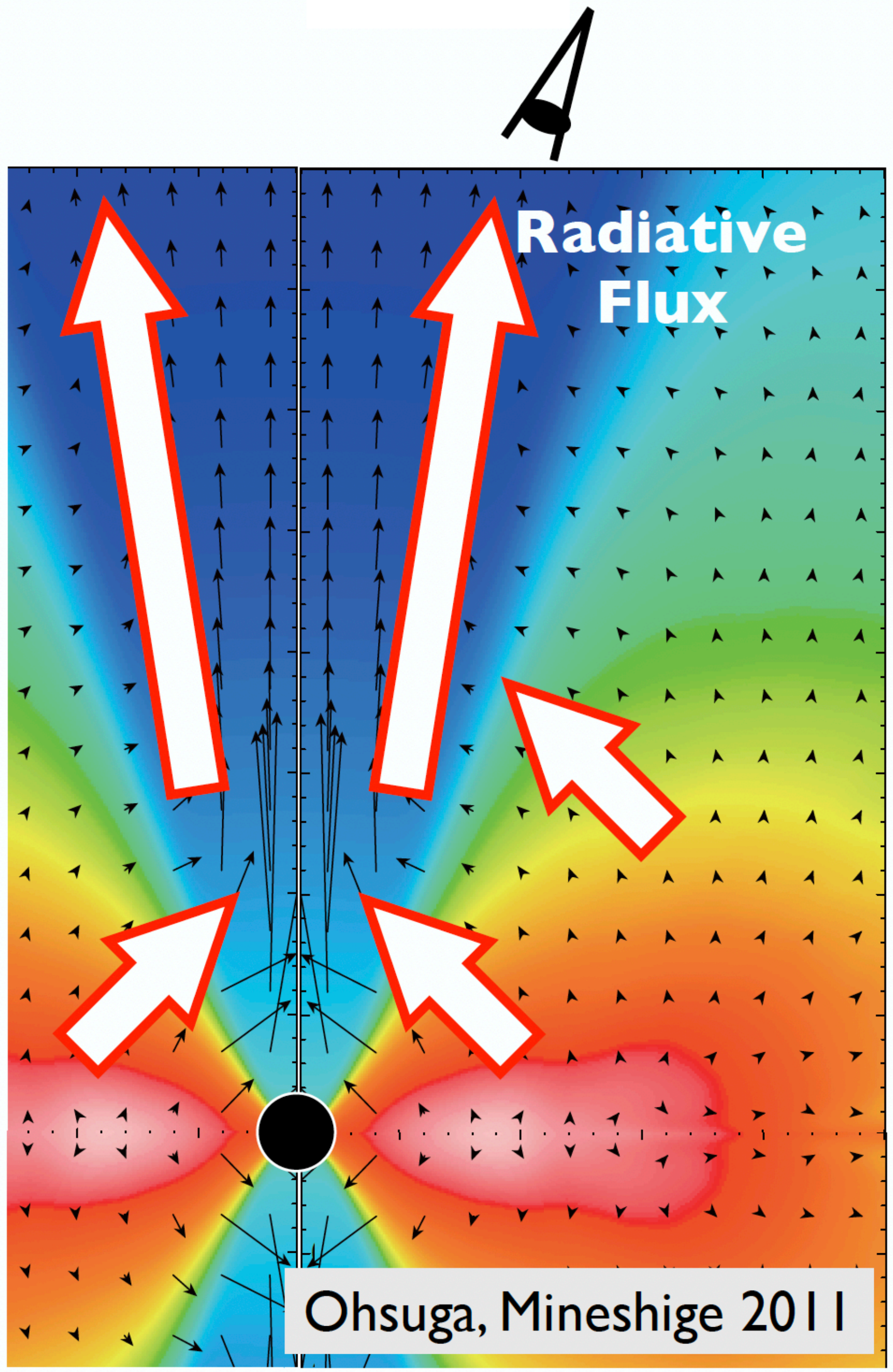
✓ The radius of the disk is $R_{\rm in} = 100\ r_{\rm g}$; at $R_{\rm in} = 100\ r_{\rm g}$, the disk becomes supercritical for $\dot{m}_{\rm in} = 25$.

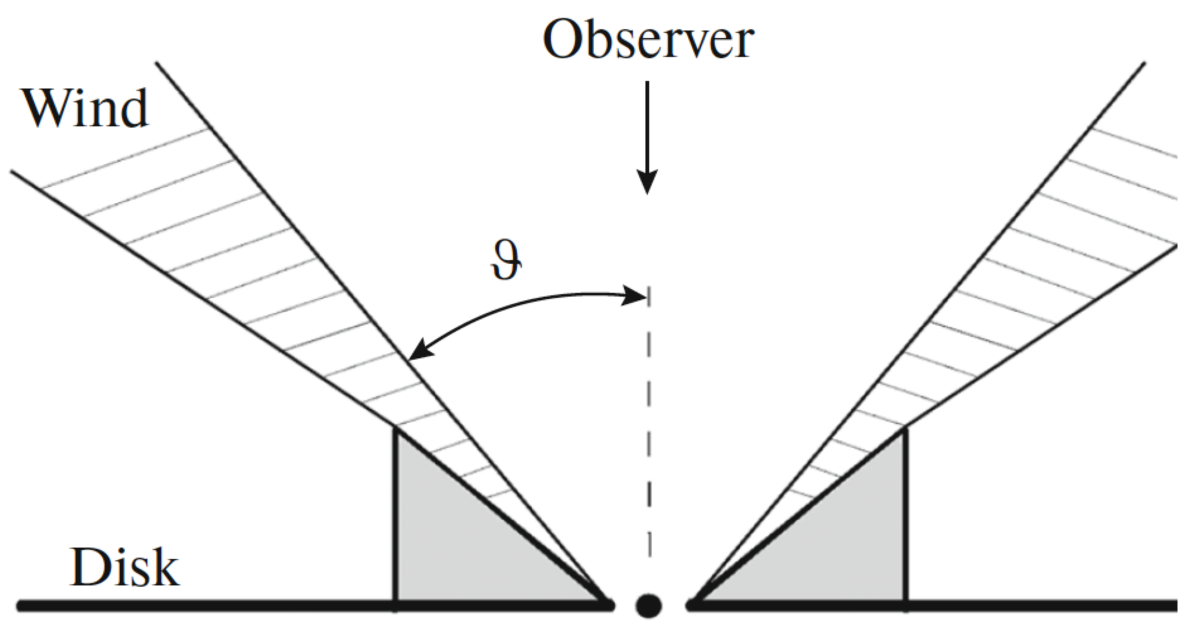


Maximum velocity at the surface $0.1\ c \le v_z \le 0.2\ c$, in the inner region of the disk $3.8\ r_g \le R \le 8 r_g$.

Future work

- Determine the radial structure of the supercritical accretion disk, taking into account advection and photon trapping.
- Ultra luminous X-Ray Sources: extragalactic objects; X-ray luminosities exceed the Eddington luminosity for a typical stellar-mass black hole.





Fabrika et al. 2021

- The radiative flux is mildly collimated since the disk is optically and geometrically thick (observed luminosity sensitive to observer's viewing angle).

- Apparent luminosity becomes highly super Eddington for face-on observers. Large luminosity of ULXs ($> 10^{39-40}$ erg/s) can be explained for the face-on case.

Thanks

Back up slides

Outflow from super-Eddington flow: where it originates from and how much impact it gives?
Takaaki KITAKI , Shin MINESHIGE, Ken OHSUGA and Tomohisa KAWASHIMA (2021)
PASJ 2021, ArXiv: arXiv:2101.11028v1

2D-RHD simulation of the super-Eddington accretion flow in a large calculation box,adopting a very large initial Keplerian radius. Newtonian gravity.

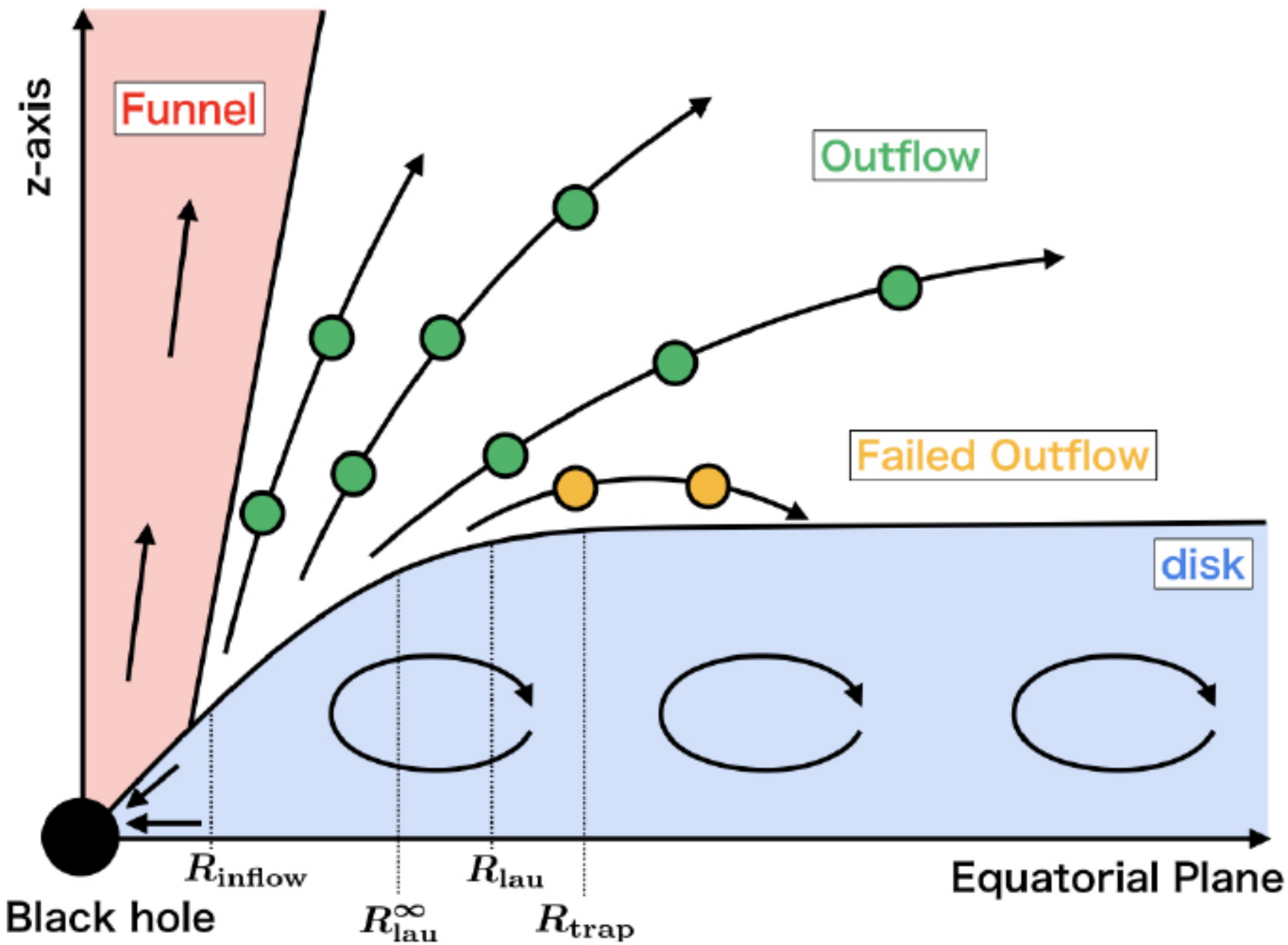


Fig. 15. Schematic view of the structure of the super-Eddington accretion flow and associated outflow based on our numerical results. The black arrows indicate the gas motion.

Disk thickness (H) proportional to R close to the black hole $H/R \approx 1$

Lauching region
$R_{\rm lau} \approx 0.75\ \dot{m} r_s$

Super-Eddington accretion disks around Supermasive Black Holes
Jiang at al. 2019
Global three-dimensional radiation magnetohydrodynamical simulations

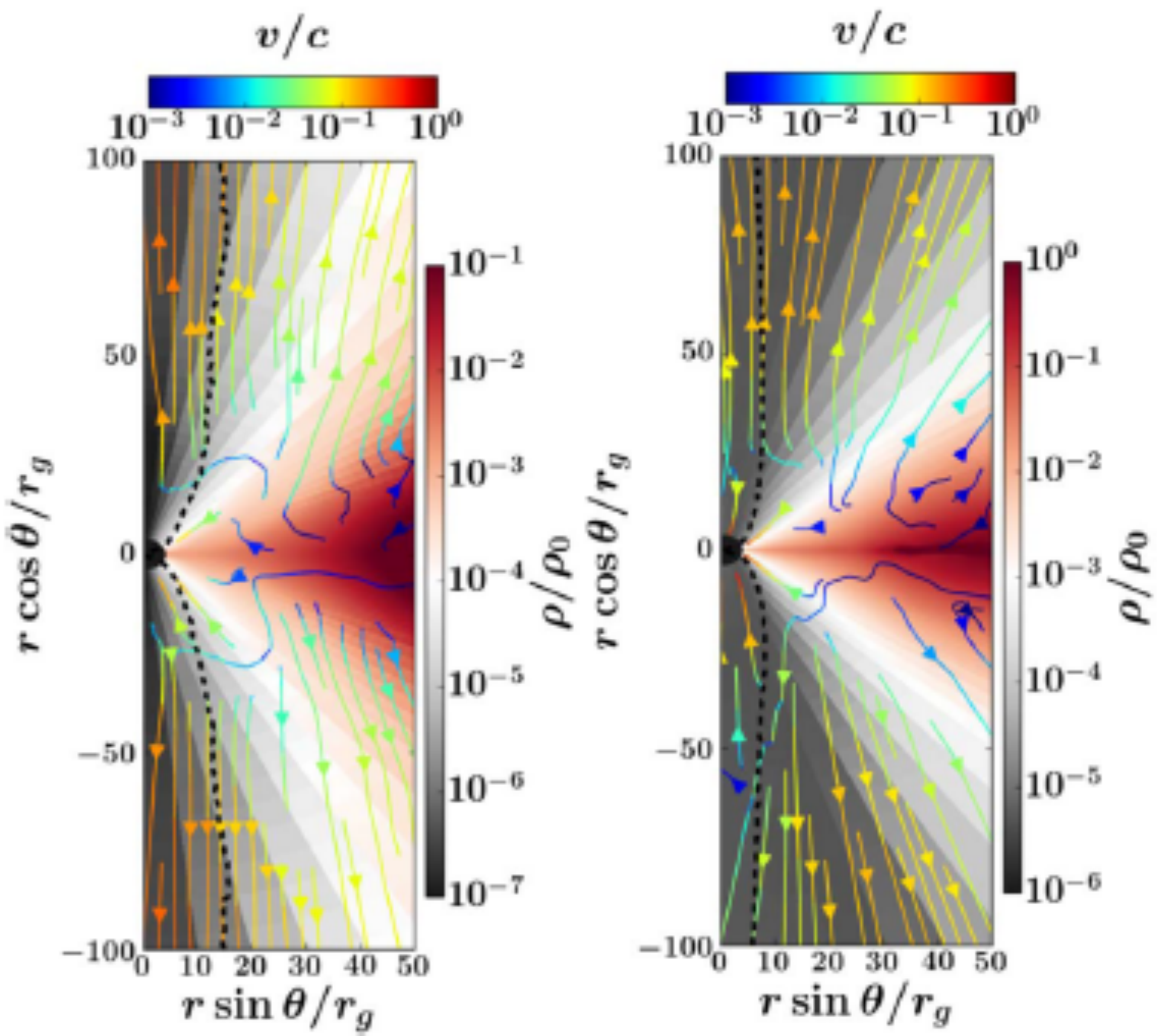


Figure 5. Time and azimuthally averaged spatial structure of density ρ (color) and mass-weighted flow velocity v_r, v_θ (streamlines) at the inner regions of the disks. The color of the streamlines represents the velocity magnitude $v \equiv \sqrt{v_r^2 + v_\theta^2}$. From left to right, they are for the two runs AGNB25 and AGNB52. The dashed black lines indicate the locations where the integrated optical depth from the rotation axis along the horizontal direction is 1.

Magnetic field

The Astrophysical Journal, 736:2 (18pp), 2011 July 20 doi:10.1088/0004-637X/736/1/2

GLOBAL STRUCTURE OF THREE DISTINCT ACCRETION FLOWS AND OUTFLOWS AROUND BLACK HOLES FROM TWO-DIMENSIONAL RADIATION-MAGNETOHYDRODYNAMIC SIMULATIONS

Ken Ohsuga[1,2] and Shin Mineshige[3]
[1] National Astronomical Observatory of Japan, Osawa, Mitaka, Tokyo 181-8588, Japan
[2] School of Physical Sciences, Graduate University of Advanced Study (SOKENDAI), Shonan Village, Hayama, Kanagawa 240-0193, Japan
[3] Department of Astronomy, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan
Received 2010 October 29; accepted 2011 April 21; published 2011 June 27

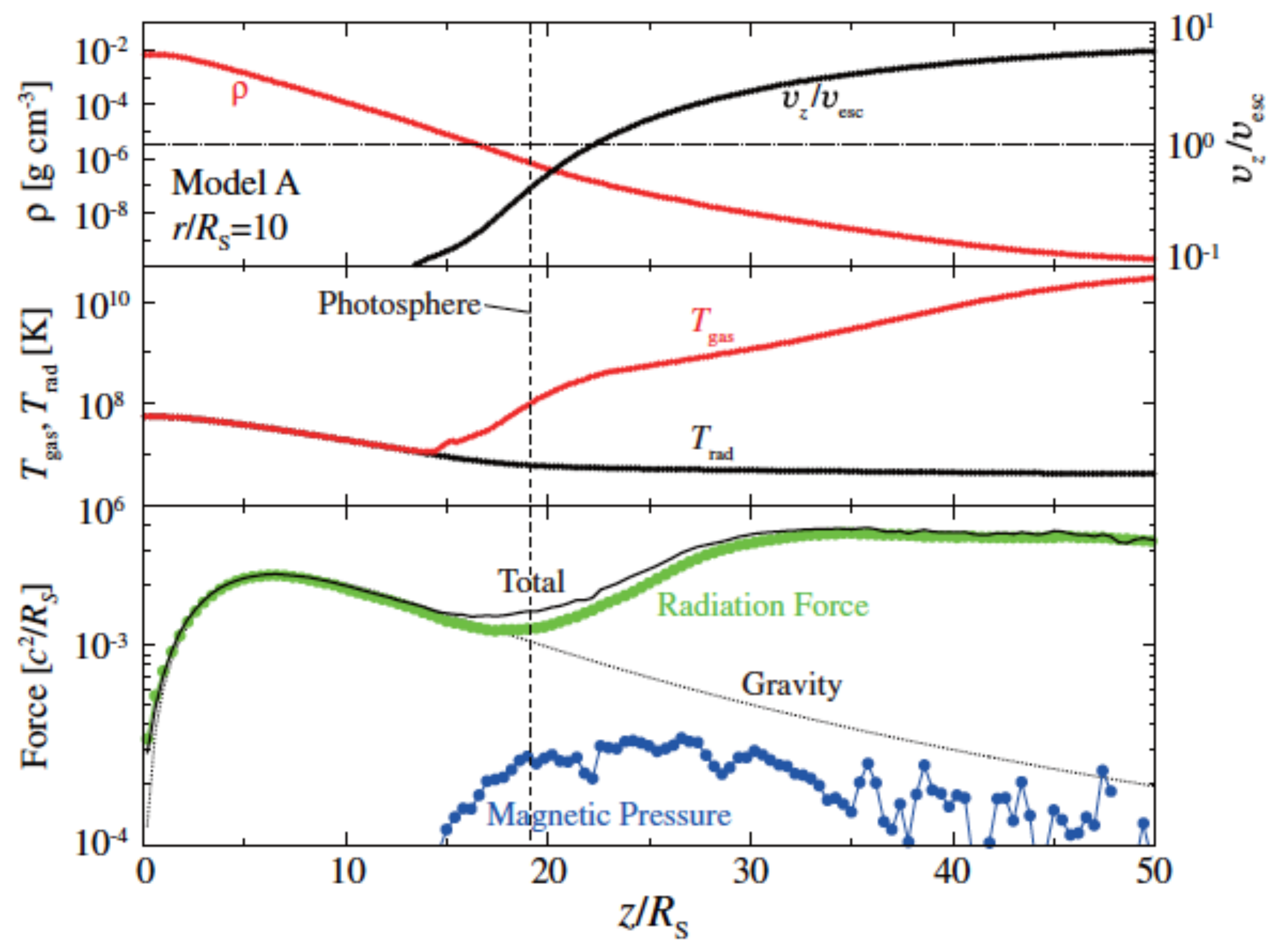


Figure 6. Vertical structure at $r = 10\,R_S$ for model A. Top panel: the density (red) and vertical component of the velocity normalized by the escape velocity (black). Middle panel: the gas and radiation temperatures. Bottom panel: the vertical components of the gravity (dotted line), the radiation force (green filled circle), and the magnetic-pressure force (blue filled circle). The sum of the vertical forces due to the radiation, the magnetic pressure, the magnetic tension, and the gas pressure is plotted by the black solid line (total). Here, the gas-pressure force (red filled circle) and the magnetic-tension force (blue open circle) are not represented, since they are so small or negative. All values are time-averaged over 6–7 s.

$$\dot{M}_{\rm acc} \sim 10^2 L_{\rm E}/c^2$$

Diffusion approximation: medium optically thick and radiation isotropy.

The radiation transfer in the vertical direction takes the expression

$$f_{\mathrm{rad}}(z) = -\frac{4\sigma}{3\kappa_T \rho(z)} \frac{\partial T^4(z)}{\partial z} = -\frac{4\sigma}{3} \frac{\partial T^4(z)}{\partial \tau}. \tag{209}$$

Here, σ is the Stefan-Boltzmann constant, κ_T denotes the electron scattering opacity, f_{rad} is the radiation flux in the vertical direction and T is the temperature of the gas in the disk.

We assume that the power added into the radiation flux is proportional to the gas density in the disk:

$$\frac{df_{\mathrm{rad}}}{dz} = \frac{\rho(z)}{\bar{\rho} H} f_{\mathrm{rad}}(H), \tag{210}$$

or alternatively,

$$f_{\mathrm{rad}}[\tau(z)] = \frac{\tau(z)}{\tau(H)} f_{\mathrm{rad}}(H). \tag{211}$$

The relation between the pressure and density of the disk in the vertical direction is polytropic, that is, $p = K\rho^\gamma$, where γ is the polytropic index. Using Eq. (217), we obtain[3]

$$\rho(\tau) = \left(1 + 3\frac{\tau(H)}{8} - 3\frac{\tau^2(z)}{8\tau(H)}\right)^{1/\gamma}. \tag{218}$$

Photon trapping

In slim disk models photon trapping is treated as advection of photon entropy.

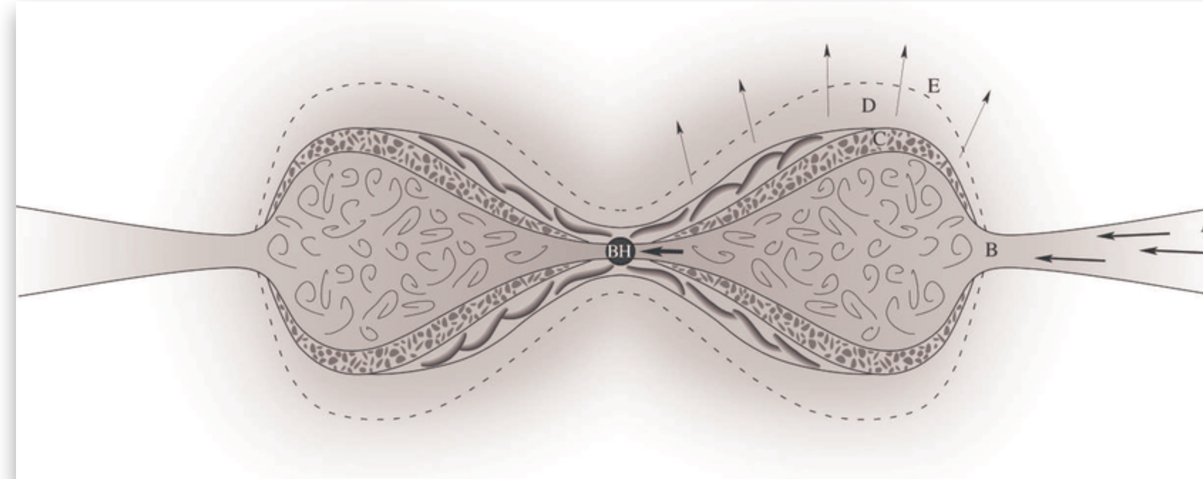
The Astrophysical Journal, 574:315–324, 2002 July 20

DOES THE SLIM-DISK MODEL CORRECTLY CONSIDER PHOTON-TRAPPING EFFECTS?

Ken Ohsuga and Shin Mineshige
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
Masao Mori
Institute of Natural Science, Senshu University, Kawasaki, Kanagawa 214-8580, Japan
and
Masayuki Umemura
Center for Computational Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan
Received 2001 December 30; accepted 2002 March 21

In the slim disk models, the vertically integrated approximation is adopted: $F^z_{surf} \approx \sigma T_c^4/\tau$. This is only valid when the vertical diffusion time scale is shorter than the accretion time scale. How to treat this problem? Solve radiative transfer equation.

Supercritical disks



When the accretion regime is supercritical, the disk is divided into two regions: an outer standard disk without winds, and an inner, radiation-dominated disk that accretes matter at the Eddington rate and ejects the excess of gas in the form of a radiation-driven wind.

$r_{\rm cr} = \frac{9\sqrt{3}\sigma_{\rm T}}{16\pi m_{\rm p} c}\dot{M} \approx 4\dot{m} r_{\rm g}$ is the critical radius where the transition occurs.

$L_{\rm bol} = L_{\rm Edd}\frac{2}{3\sqrt{3}}\left(1 + \ln\left(4\dot{m}\frac{r_{\rm g}}{r_{\rm in}}\right)\right)$ is the disk luminosity.

$v_{\rm w} \sim \sqrt{\frac{GM_{\rm BH}}{r_{\rm cr}}} = \frac{c}{2\sqrt{\dot{m}}}$ is the wind velocity.

$$\tau_{\rm ph} = -\int_{\infty}^{z_{\rm ph}} \Gamma_{\rm w}\left(1 - \beta\cos\theta\right)\kappa_{\rm co}\rho_{\rm co}{\rm d}z = 1$$

determines the photosphere of the wind.