

# Magnetization of Relativistic Current- Carrying Jets with Radial Velocity Shear

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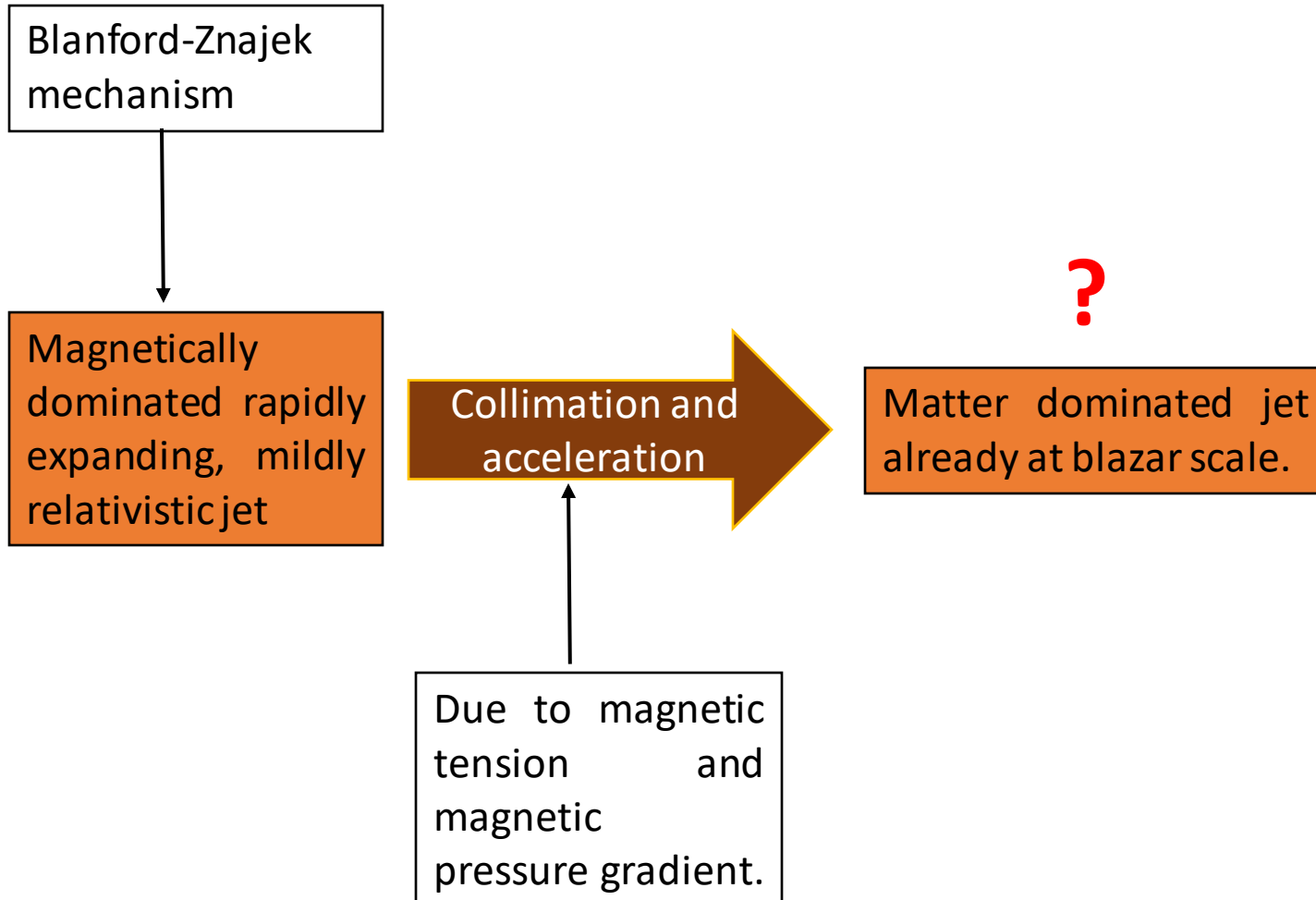
# Astrophysical jets at different scales

Blanford-Znajek  
mechanism

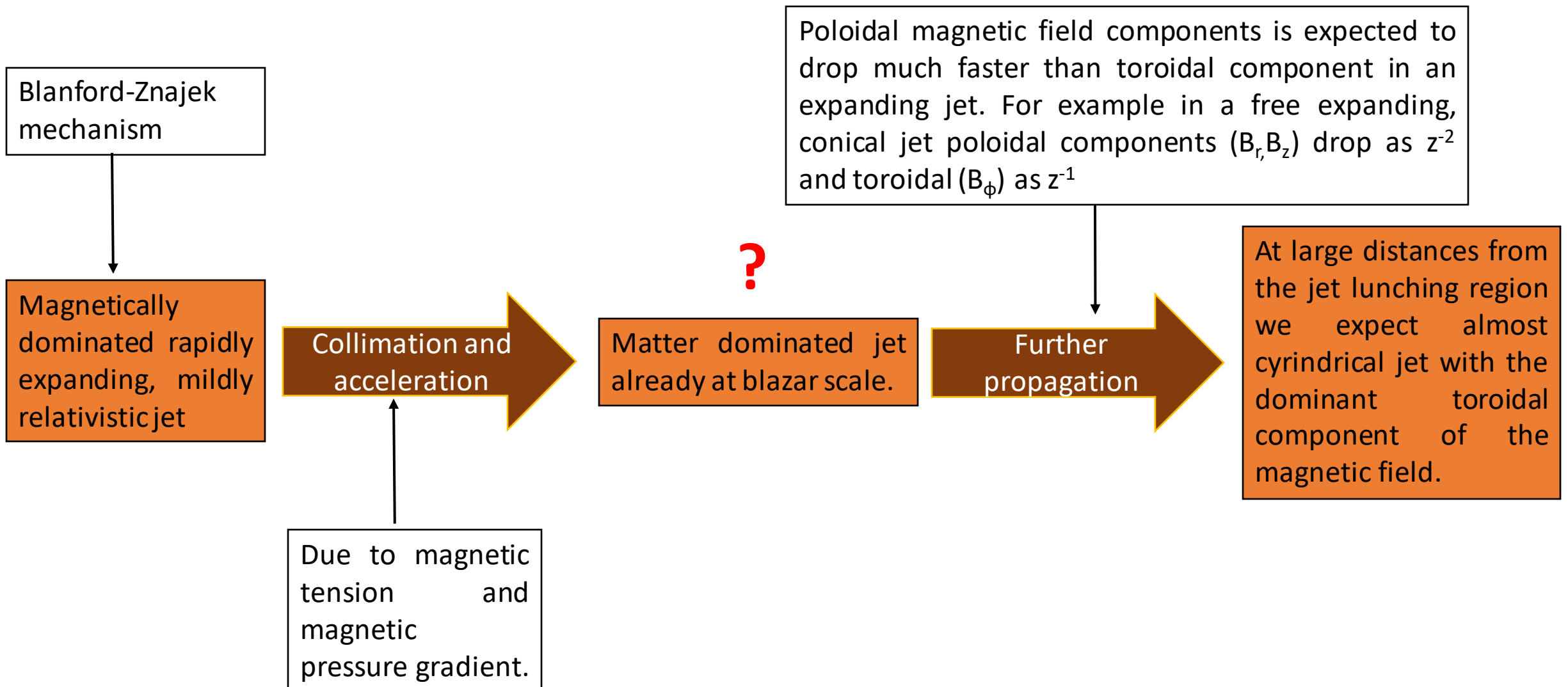


Magnetically  
dominated rapidly  
expanding, mildly  
relativistic jet

# Astrophysical jets at different scales



# Astrophysical jets at different scales



# Jet model assumptions

## Geometry and profiles:

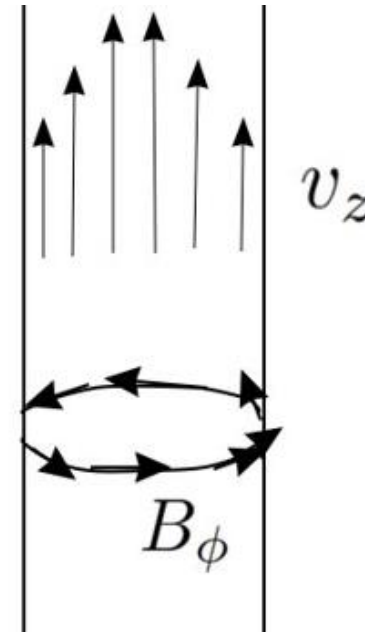
- Cylindrical axisymmetric geometry.
- Radial velocity shear.
- Non-rotating.
- Magnetic fields has only toroidal component.

## Physical constrains:

- Magnetohydrostatic equilibrium within the jet, pressure equilibrium with the external medium.

## Equation of state:

- Ultra-relativistic equation of state.



How does it translate to equations?

## MHD force equation

$$\rho \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = -\nabla P + \mathbf{j} \times \mathbf{B} \xrightarrow{J' \propto \nabla \times B'} \partial_r P = -\frac{1}{8\pi r^2} \partial_r \left( \frac{r^2 B_\phi^2}{\Gamma^2} \right)$$

Any spatial changes in the magnetic pressure across the jet must be counterbalanced by the changes in the particle pressure and the magnetic tension

## MHD force equation

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$$J' \propto \nabla \times B'$$

$$\partial_r P = -\frac{1}{8\pi r^2} \partial_r \left( \frac{r^2 B_\phi^2}{\Gamma^2} \right)$$

Any changes in the magnetic pressure across the jet must be counterbalanced by the changes in the particle pressure and the magnetic tension

For a given magnetic field and velocity profile we can calculate pressure profile

## Parametrisation

Magnetic field and pressure normalization:

$$x \equiv r/R_j$$

$$b(x) \equiv \frac{B_\phi(x)}{B_\phi(1)} : \quad b(0) = 0, \quad b(1) \equiv 1$$

$$p(x) \equiv \frac{P(x)}{P(1)} : \quad p(0) > 0, \quad p(1) \equiv 1$$

$$f(x) \equiv \frac{b^2(x)}{\Gamma^2(x)}$$

Normalized rest-frame magnetic pressure



# Magnetization

$$L_p = 8\pi c \int dr r \beta \Gamma^2 P$$

Energy flux associated with jet particles.

$$L_B = \frac{1}{2} c \int dr r \beta B_\phi^2$$

Poynting flux.

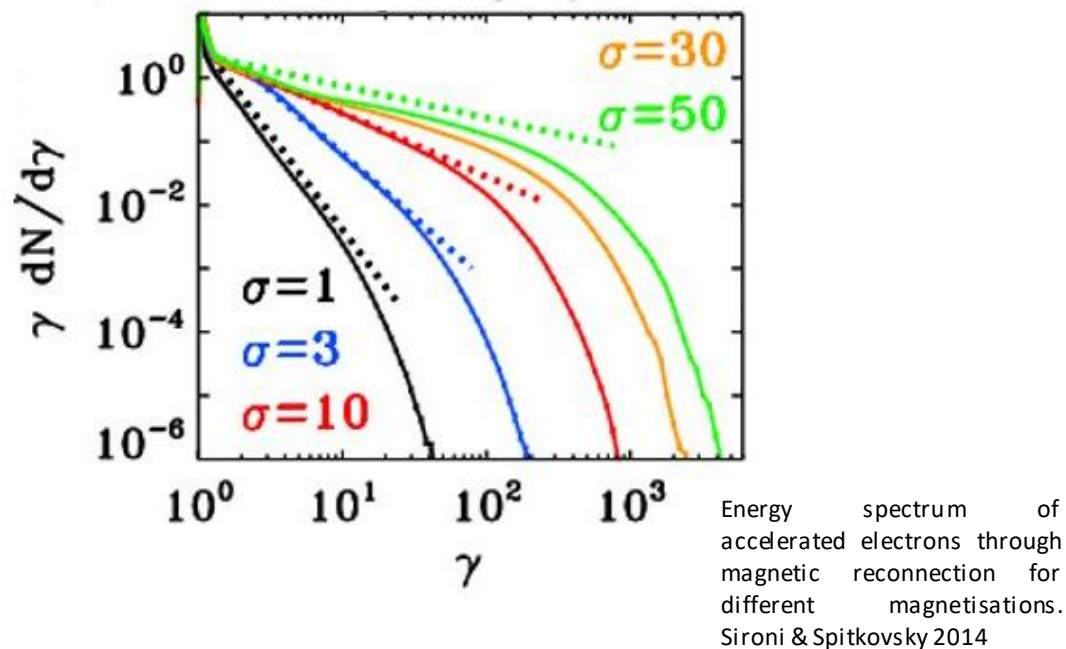
$$\sigma \equiv \frac{L_B}{L_p}$$

Magnetisation.

# Magnetisation effect on particles acceleration mechanism

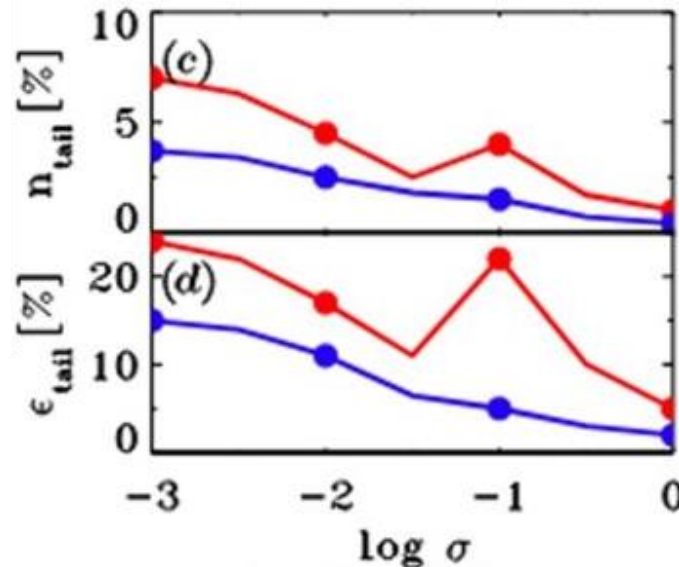
## Magnetic reconnection

Increasing magnetisation increases efficiency of particles acceleration.



## Shocks

Increasing magnetisation causes decrease of the shock compression rate, what makes shock acceleration much less efficient.



Fraction of ions (red) or electrons (blue) in the nonthermal tail; (d) fraction of energy in the ion (red) or electron (blue) nonthermal tail, with respect to the total kinetic energy of that specie  
Sironi & Spitkovsky 2011

# Definition of the model

We define:

- Magnetic field profile
- Velocity profile
- $q$  parameter value

Calculations

$$p(x) = 1 + q - q f(x) + 2q \int_x^1 ds \frac{f(s)}{s}$$

$$\sigma = \frac{q \int_0^1 dx x \beta(x) \Gamma^2(x) f^2(x)}{2 \int_0^1 dx x \beta(x) \Gamma^2(x) p(x)}$$

$$\beta_{pl}^{-1}(x) \equiv \frac{P_B(x)}{P(x)} = \frac{q f(x)}{p(x)}$$

We calculate:

- Pressure profile.
- Magnetization.
- $\sigma$  and  $\beta_{pl}$  profiles



# Motivations

Observation of Lisanti & Blandford (2007): for the jet model setup as considered here, numerical solutions to the jet magnetization parameter always return  $\sigma < 1$ .

We wanted to:

- Find an analytical proof for this.
- Explore local changes of magnetisation and beta parameter.

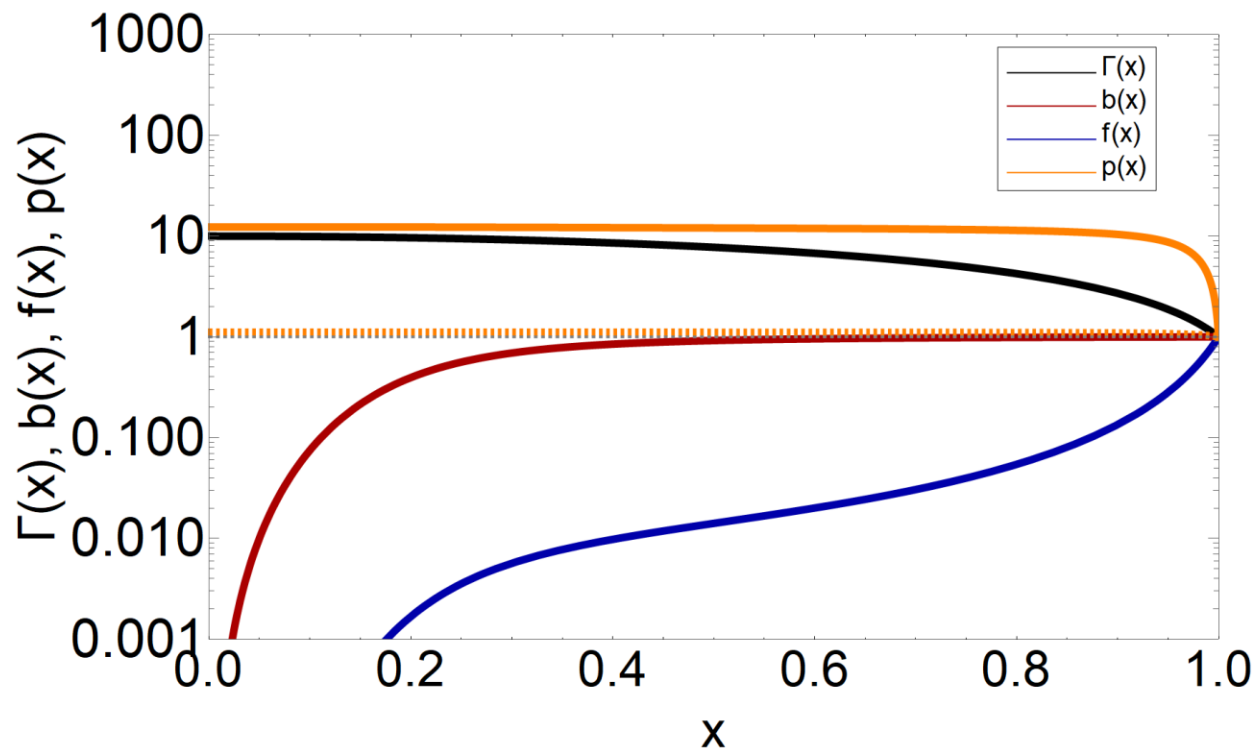
# Proof of low magnetization for a class of models

- Assumptions on  $f(x)$  - comoving magnetic field square:
  - $f(x)$  is continuous.
  - $f(x)$  maximal value is reached on the jet boundary:  $f(1) = 1$ .
- Result:
  - For every profile magnetisation parameter is:

$$\sigma < \frac{1}{2}$$

# Other profiles

1. Follows the assumptions of proof.

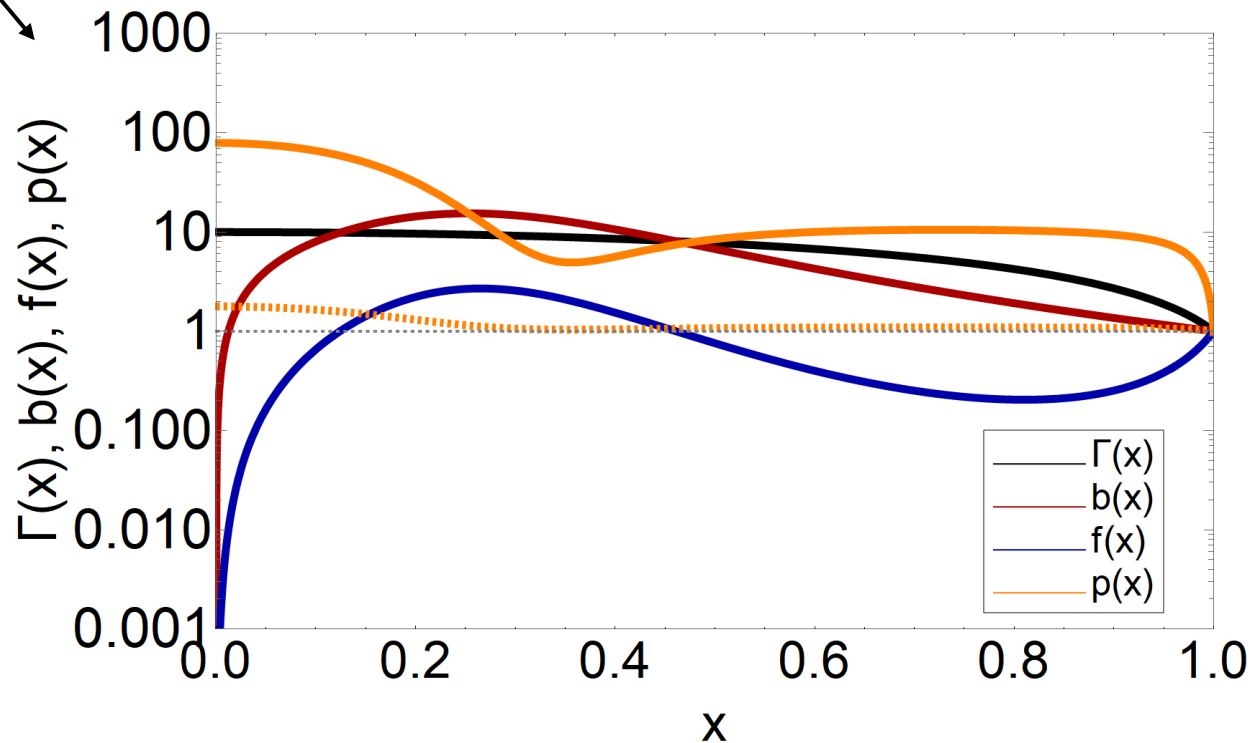
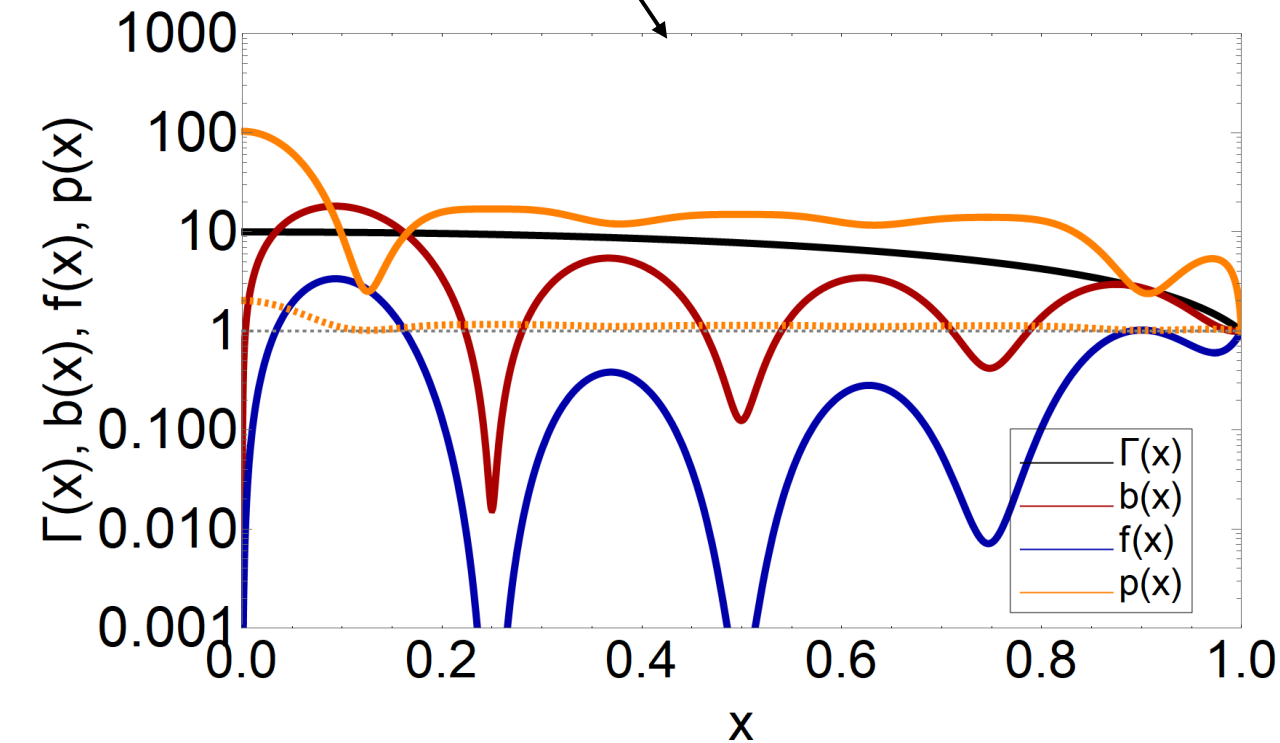
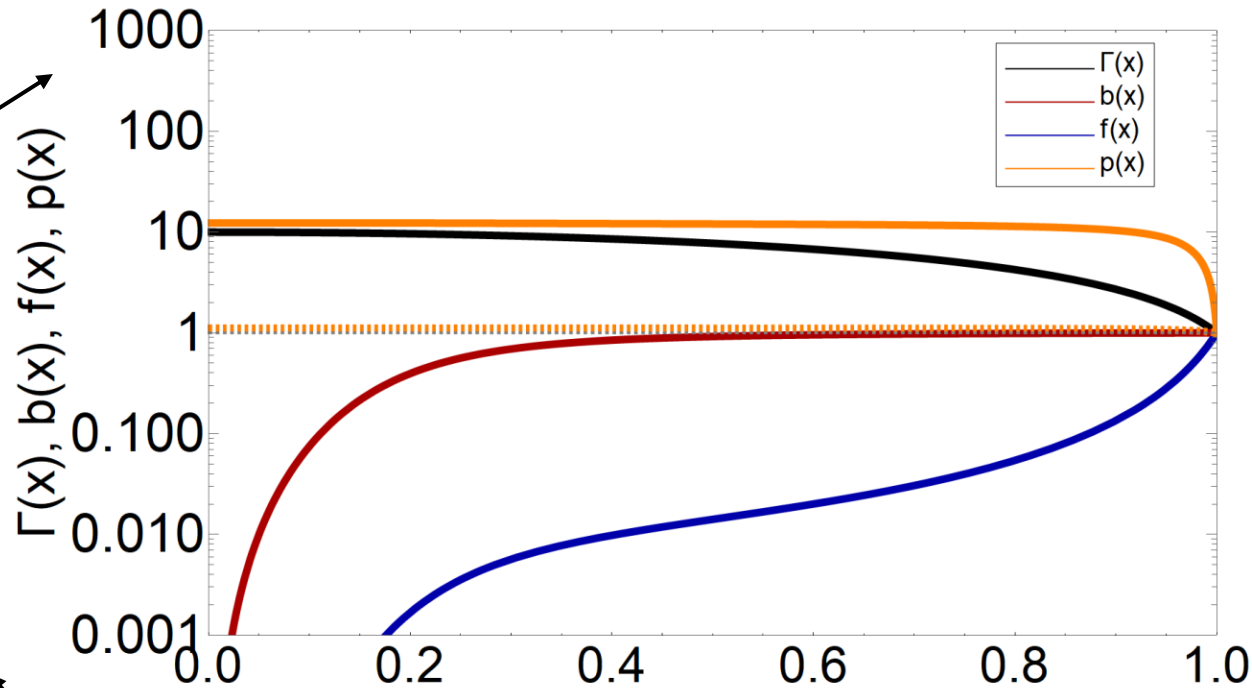


# Other profiles

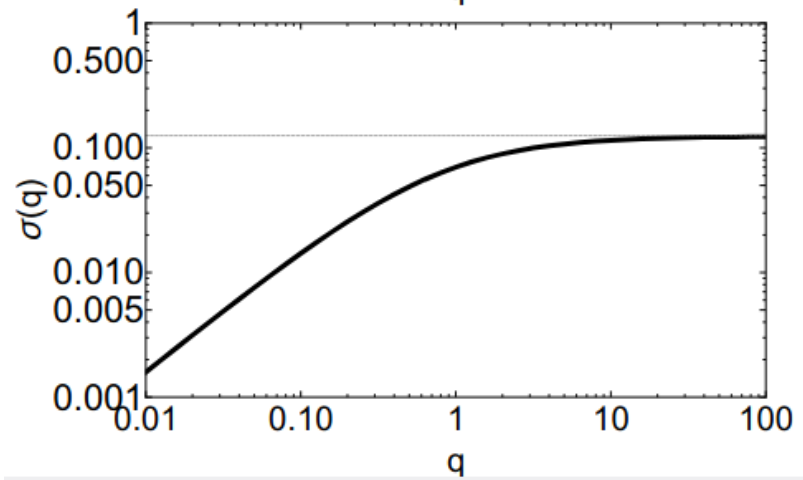
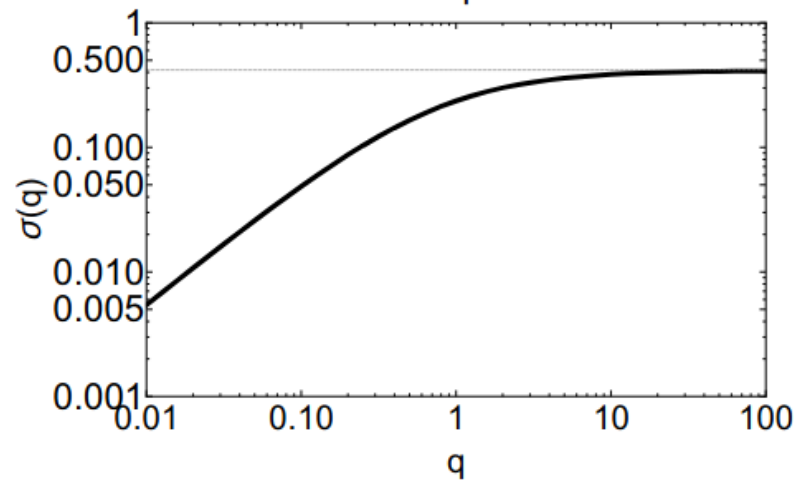
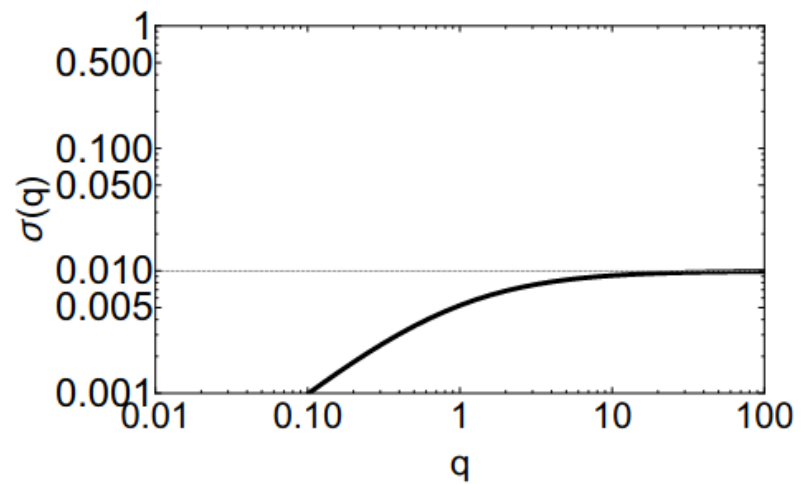
1. Follows the assumptions of proof.

2. Does not follow the assumptions of proof, with one maximum of  $f(x)$ .

3. Does not follow the assumptions of proof, with multiple maxima of  $f(x)$ .



Magnetization  
as a function  
of  $q$   
parameter



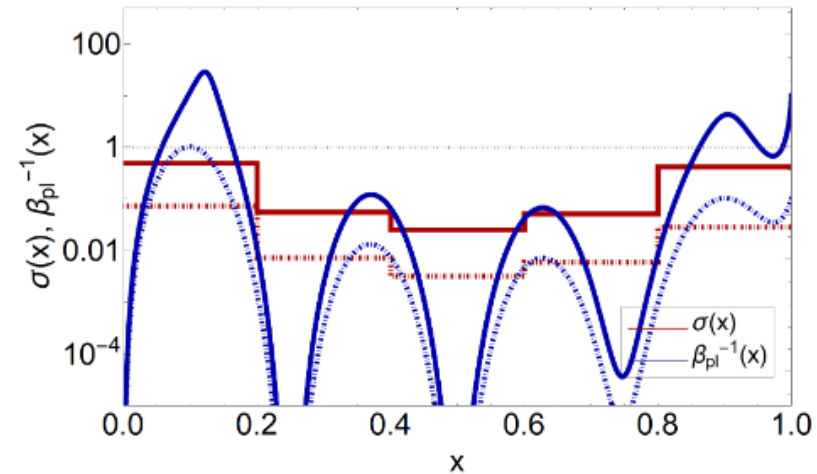
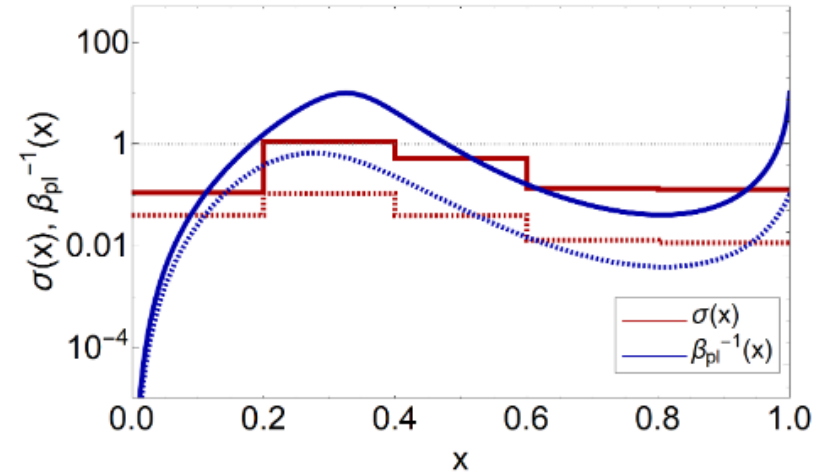
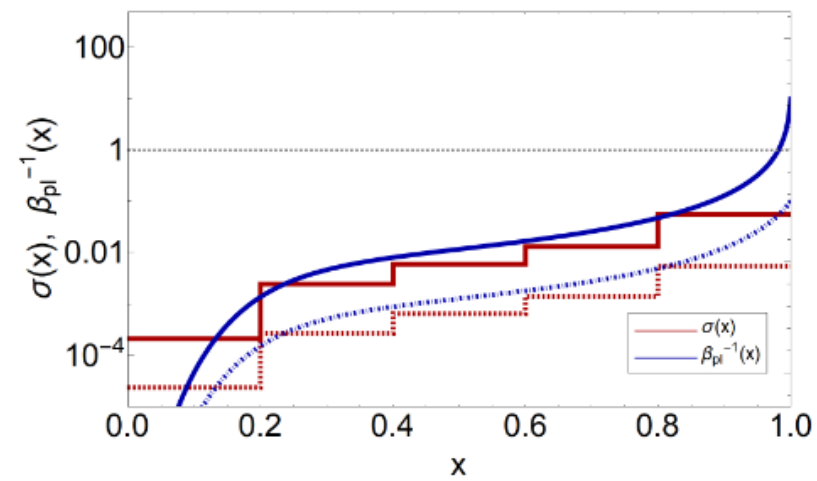


# Stratification in magnetisation

In this very simple jet model, we are able to obtain stratification in local magnetization and in the value of the beta parameter.

Magnetization has a crucial influence on the properties of the shocks and magnetic reconnection as well as on accelerated particles energy spectrum.

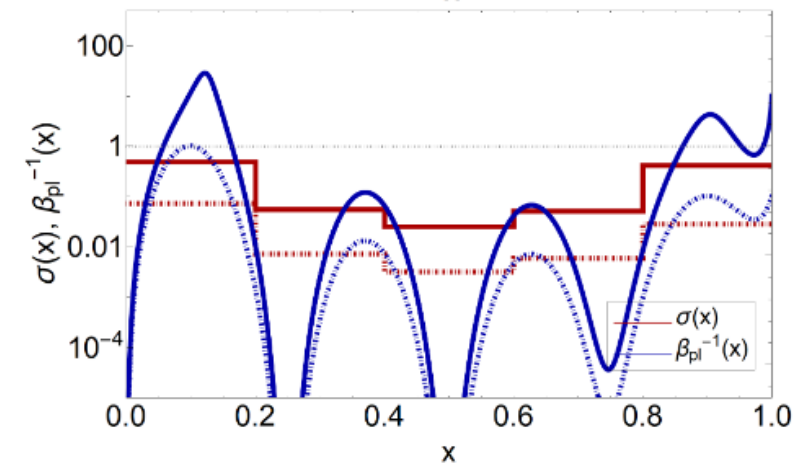
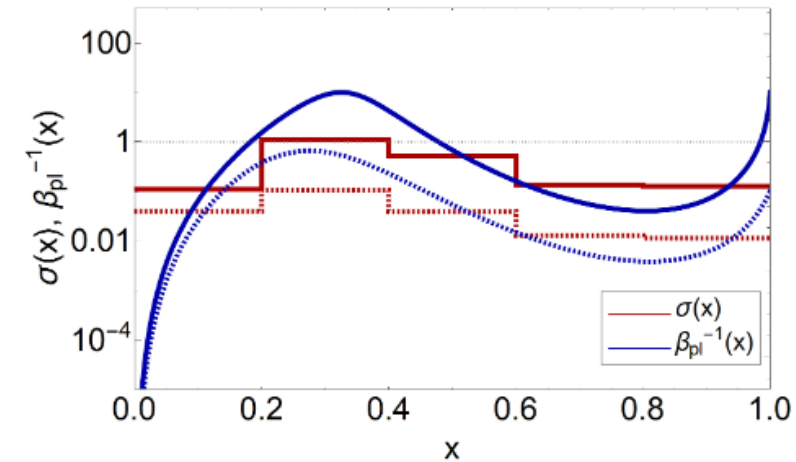
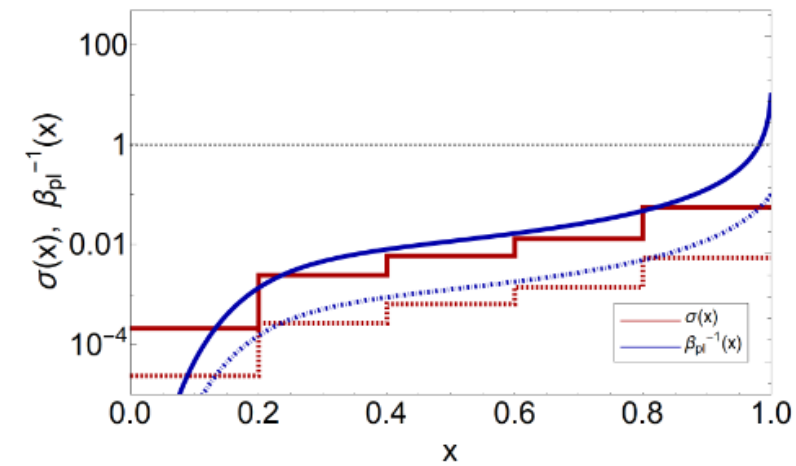
Due to stratification in magnetisation, depending on the layer different mechanism may dominate.



In this very simple jet model, we are able to obtain stratification in local magnetization and in the value of the beta parameter.

Due to velocity stratification Doppler boosting effect will be the strongest for different layers of the jet depending on viewing angle

Depending on the viewing angle we may observe regions with different magnetizations and therefore different dominating acceleration mechanism and its properties.



Is it possible to get  
magnetically dominated  
jet within the framework  
of this simple model?

Yes.

**BUT** at the expense of:

- Huge pressure gradients are required  
( -> discontinuity)
- No velocity shear
- Maximum magnetizations at most of the order  $\sim$  few

To obtain  $\sigma \sim 5$  we need a pressure drop of the order of  $10^7$

# Summary

Król et al. 2022, ApJ

Assumptions on jet model: cylindrical axisymmetric geometry, radial velocity shear, non-rotating. MHD equilibrium within the jet, ultra-relativistic equation of state.

We analytically proved that jet is always matter-dominated if the maximal value of the normalized rest-frame magnetic pressure is at the jet boundary.

For jet to be Poynting flux dominated we need huge gradient of the gas pressure (or discontinuity) and magnetisation parameter is still  $< O(10)$

For relatively simple profiles we obtain stratification in magnetisation parameter may result in much different properties of the jet depending on layer. Together with stratification in velocity and Doppler effect it may result in heavy dependence on observed characteristics on viewing angle

Plans for future:

- Checking stability of the stratification in magnetic field with RMHD simulations.
- We study radiation and polarisation of this kind of jet.









Backup slides

# Stability and poloidal magnetic field

In the absence of velocity shear, jets with purely toroidal magnetic field are known to be susceptible to current-driven instability. Adding  $B_z$  component may stabilize the jet.

How adding  $B_z$  component affects our results?

If  $B_z$  is uniform across the jet ( $\partial_r B_z = \partial_z B_z = \partial_\phi B_z = 0$ ) the results are unchanged since no additional current component appears ( $J' \propto \nabla \times B'$ ) and therefore magnetohydrostatic equilibrium condition does not change ( $\nabla' P \propto J' \times B'$ ). The Poynting flux on the other hand is increased only in  $\phi$  direction (so it does not add to Poynting flux in along the jet).

In the presence of radial gradient ( $\partial_r B_z \neq 0$ ) the situation changes. Additional current component appears and therefore pressure profile changes. However, configurations with narrow jet spine tend to be in equipartition (Lyubarsky 2019). Moreover, configurations with poloidal jet spine and dynamically important toroidal magnetic field in the outer part of the jet are unstable (Mizuno et al. 2012).

# Magnetohydrodynamics approximation

- Fluid approximation:
  - We can describe plasma as a fluid (with local thermodynamical quantities)
  - Time scale of the change of thermodynamical parameters  $\gg$  timescale of processes at kinetic scale
- Plasma is electrically neutral: length scale  $\gg$  Debye length
- Ideal MHD: plasma is perfectly conductive, all dissipative processes are neglected.

# Introducing parametrization

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Total magnetization

$$\sigma = \frac{q}{2} \frac{\int_0^1 dx x \beta(x) \Gamma^2(x) f^2(x)}{\int_0^1 dx x \beta(x) \Gamma^2(x) p(x)}$$

Local magnetization

$$\sigma(x \pm \Delta) \equiv \frac{\langle f(x) \rangle_{\Delta}}{\langle p(x) \rangle_{\Delta}} \equiv \frac{q}{2} \frac{\int_{x-\Delta}^{x+\Delta} dy h(y) f(y)}{\int_{x-\Delta}^{x+\Delta} dy h(y) p(y)}$$

Ideal MHD equations  
(non-relativistic)

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{V}) = 0,$$

Conservation of mass.

Euler equation

$$\rho \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \nabla) \mathbf{V} \right] = -\nabla P + \mathbf{j} \times \mathbf{B}$$

"Forces" = pressure gradient + magnetic tension and gradient of magnetic pressure.

Entropy equation

$$\frac{d}{dt} \left( \frac{P}{\rho^\gamma} \right) = 0$$

Entropy is conserved along the stream line, d/dt - material derivative.

Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B})$$

The magnetic field is frozen into the fluid and has to move along with it.