A 4th-order accurate finite volume method for ideal and resistive classical and special relativistic MHD in the PLUTO code

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ABSTRACT

We present a novel implementation of a genuinely 4th-order accurate finite volume scheme for multidimensional classical and special relativistic magnetohydrodynamics (MHD) in both ideal [1] and resistive [2] regimes based on the constrained transport (CT) formalism in the PLUTO code [3]. Our scheme is rooted over the method originally proposed by McCorquodale and Colella [4] but introducing several novel aspects when compared to its predecessors, yielding a more efficient computational tool. Among the most relevant ones, our scheme exploits pointwise to pointwise reconstructions (rather than one-dimensional finite volume ones), employs all the generalized upwind averaging techniques of the UCT method of Mignone and Del Zanna [5], with the addition of a new relativistic UCT-GFORCE average, and ensures robustness through sophisticated limiting strategies that include both a discontinuity detector and an order reduction procedure. Such method has so far produced results in numerical simulations that are unfeasible with traditional low order schemes.

THE FINITE VOLUME FORMALISM IN (RELATIVISTIC) MHD

Our method is targeting the equations of ideal and resistive (R)MHD in Cartesian geometry. 1) Hyperbolic sub-system for the evolution of the conservative flow variables:

\[
\frac{\partial \mathbf{V}}{\partial t} + \nabla \cdot \mathbf{F} = 0, \quad \mathbf{F}(\rho, \mathbf{v}, \mathbf{B}, \rho_\parallel, \mathbf{v}_\parallel, \mathbf{B}_\parallel) = \begin{pmatrix}
\rho \mathbf{v} \\
\rho \mathbf{v} \cdot \mathbf{v} \\
\mathbf{B} \times \mathbf{v} \\
\rho \mathbf{v}_\parallel \\
\rho \mathbf{v}_\parallel \cdot \mathbf{v}_\parallel \\
\mathbf{B}_\parallel \times \mathbf{v}_\parallel
\end{pmatrix}
\]

2) Induction equation for the evolution of the magnetic field:

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0
\]

endowed with the solenoidal condition \( \nabla \cdot \mathbf{B} = 0 \) satisfied at \( t = 0 \) and preserved \( \forall t > 0 \).

THE 4TH-ORDER NUMERICAL SCHEME (IN BRIEF)

High order Godunov-type High-Resolution-Shock-Capturing scheme based on Reconstruct-Solve-Average strategy (for the details of the scheme see [1] and [2]).

1) Conversion step using Laplacian operators \( \Delta [4] \) to obtain point values at cell centers \( \mathbf{V}_c \).

2) Pointwise Reconstructions [1] reconstruct \( \mathbf{V}_c \) at interfaces without having to average \( \mathbf{V}_c = (\mathbf{V}_e + \mathbf{V}_w)/2 \).

3) Limiter [1] introduction of 2 types of discontinuity detectors \([8,9]\) and an order reduction procedure.

4) Upwind Constrained Transport + all the generalized upwind EMF averages \([8]\) + punctual reconstructions.

RESULTS, CURRENT AND FUTURE DEVELOPMENT

- Genuinely 4th-order accurate finite volume scheme for both ideal and resistive (R)MHD accounting several innovative aspects that yield an accurate, robust and efficient computational tool.
- Introduction of pointwise to pointwise reconstruction operators that ease up the structure of the scheme.
- Robustness assured by a limiter that allows the 4th-order scheme to carry out extremely severe tests yielding unprecedented results.
- Generalization of the UCT-GFORCE average to relativistic MHD.

REFERENCES